MACHINE DESIGN

INTERNATIONAL
TEXTBOOK COMPANY
SCRANTON, PA.

Machine Design

BOOK II

MACHINE DESIGN
Parts 3–4

225
Published by
INTERNATIONAL TEXTBOOK COMPANY
SCRANTON, PA.

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Printed in U. S. A.

INTERNATIONAL TEXTBOOK PRESS
Scranton, Pa.

98338

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MACHINE DESIGN

Serial 997C (PART 3) Edition 1

DESIGN OF JOURNALS AND BEARINGS

JOURNALS

END JOURNALS

1. Journals are the cylindrical portions of rotating pieces that turn within bearings and form the supports. Journals situated at or near the ends of a shaft, axle, or other rotating piece are termed *end journals*. Any journal situated between two end journals is called a *neck journal*.

The ordinary form of an **end journal** is shown in Fig. 1, and consists simply of a turned down part of the shaft, shoulders

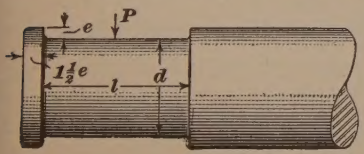

FIG. 1

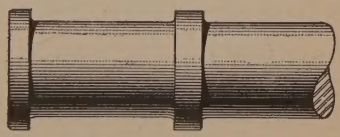

FIG. 2

being produced in this manner at each end of the journal, to prevent end play in the bearing. Sometimes, the same result is obtained without reducing the diameter of the shaft by forging collars at each end of the journal, as shown in Fig. 2.

The length of the box, or brass, on which the journal rests, however, is often made slightly shorter than the journal, permitting a slight motion lengthwise, and thereby securing uniform wear.

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SIZE AND PROPORTIONS OF END JOURNALS

2. The chief element in the design of a journal moving slowly or intermittently is strength. When journals run constantly at considerable velocity, strength is not so important a consideration as durability and freedom from liability to heat.

The dimensions of an end journal to give sufficient strength may be calculated by considering the journal as a cantilever uniformly loaded.

Let l = length of journal, in inches;
d = diameter of journal, in inches;
P = total load on journals, in pounds;
S_s = *safe* stress of material in flexure.

Then, from *Strength of Materials*, the bending moment is $\frac{w\,l^2}{2} = \frac{P\,l}{2}$, and the resisting moment is $\frac{S_s I}{c}$, which for a circular section is

$$S_s \frac{\frac{\pi\,d^4}{64}}{\frac{1}{2}\,d} = S_s \frac{\pi\,d^3}{32}$$

Hence,
$$\frac{P\,l}{2} = S_s \frac{\pi\,d^3}{32} \qquad (1)$$

or,
$$d = \sqrt{\frac{16\,P}{\pi\,S_s} \times \frac{l}{d}} = 2.26\sqrt{\frac{P}{S_s} \times \frac{l}{d}} \qquad (2)$$

Formula **2** gives the diameter of the journal when the ratio $\frac{l}{d}$ is assumed.

EXAMPLE.—Find the diameter and length of a wrought-iron journal on which there is a load of 1,200 pounds. Assume S_s = 8,500 pounds and $\frac{l}{d} = 1.4$.

SOLUTION.—Using formula **2**,

$$d = 2.26\sqrt{\frac{1,200}{8,500} \times 1.4} = 1 \text{ in., nearly. Ans.}$$

$$l = 1.4\,d = 1.4, \text{ say } 1\tfrac{7}{16} \text{ in. Ans.}$$

3. The **bearing surface,** or **projected area,** of a journal is the length multiplied by the diameter; that is, it is the area of the projection of the journal on a plane

perpendicular to the direction of the load P. The total load on the journal divided by the projected area gives the pressure per square inch of projected area—a quantity that will be denoted by p.

Hence, $$p = \frac{P}{l\,d}, \text{ or } P = p\,l\,d \qquad (1)$$

In order that the journal may not heat, the pressure p must not exceed a certain limit determined by experience. When this pressure is too great, the oil used to lubricate the journal is squeezed out, and the journal heats rapidly.

From formula **1**, Art. **2**,

$$P = \frac{\pi\,S_s\,d^2}{16\,l}$$

From formula **1**, $$P = p\,l\,d$$

Hence, $$\frac{\pi\,S_s\,d^2}{16\,l} = p\,l\,d, \text{ or } \pi\,S_s\,d^2 = 16\,p\,l^2$$

and $$\frac{l}{d} = \sqrt{\frac{\pi\,S_s}{16\,p}} \qquad (2)$$

Substituting this value of $\frac{l}{d}$ in formula **2**, Art. **2**, and reducing,

$$d = \sqrt{\frac{4\,P}{\sqrt{\pi\,S_s\,p}}} = 1.5\sqrt{\frac{P}{\sqrt{S_s\,p}}} \qquad (3)$$

From formula **1**,

$$l = \frac{P}{d\,p} \qquad (4)$$

Formula **3** may be used to compute the diameter of a journal when the pressure p per square inch of projected area is fixed. The length may then be obtained from formula **4**.

EXAMPLE.—Compute the length and diameter of a steel journal sustaining a load of 12,000 pounds. The safe stress S_s is 14,000 pounds, and the pressure per square inch of projected area is not to exceed 750 pounds.

SOLUTION.—Using formula **3**,

$$d = 1.5\sqrt{\frac{12{,}000}{\sqrt{14{,}000 \times 750}}} = 2.89, \text{ say } 2\tfrac{7}{8}, \text{ in. Ans.}$$

Hence, $$l = \frac{P}{d\,p} = \frac{12{,}000}{2\frac{7}{8} \times 750} = 5\tfrac{9}{16} \text{ in. Ans.}$$

TABLE I

PRESSURE ON BEARINGS AND SLIDES

Kind of Journal Bearing	Pressure p per Square Inch of Projected Area Pounds
Bearings subject to intermittent load at slow speeds, such as crankpins of shearing machines	3,000
Crosshead neck journals; motion oscillating	800 to 1,400
Crosshead neck journals in locomotives	1,500 to 2,000
Crankpins (slow engines)	800 to 900
Crankpins (fast engines)	500 to 800
Crankpins (marine engines)	400 to 500
Crankpins (locomotives)	1,200 to 1,800
Crankpins (small land engines)	150 to 200
Main crank-shaft bearings (slow)	300 to 450
Main crank-shaft bearings (fast)	200 to 300
Locomotive driving-axle journal	180 to 350
Flywheel-shaft bearings	150 to 250
Eccentric straps	80 to 100
Railway-car journals	250 to 300
Locomotive and tender axle bearings	200 to 250
Line shafting on cast-iron steps (Sellers)	50
Pivots	700
Collar-thrust bearings (slow)	70 to 80
Collar-thrust bearings (fast)	50 to 70
Slides, cast iron on Babbitt metal	200 to 300
Slides, cast iron on cast iron (slow)	60 to 100
Slides, cast iron on cast iron (fast)	40
Steel or iron shaft on lignum vitæ; water lubrication	350

4. The pressure p per square inch of projected area may be taken at from 400 to 800 pounds when the journal runs constantly at a speed under 150 revolutions per minute. For journals that run slowly or intermittently, p may be much greater, while for journals that run faster than 150 revolutions per minute, the pressure p should vary inversely as the number of revolutions. That is, letting N = number of revolutions per minute, $p = \frac{a}{N}$, where a is a constant.

Another consideration affecting the allowable pressure p is the direction of the load. In some journals the load acts only in one direction, generally downwards; in others, as, for example, crankpins and crosshead pins, the direction of the load changes at every revolution. In the latter case, the pressure p may be twice as great as where the load acts in one direction only, because the change in the direction of the load permits a more perfect lubrication of the bearing. When, however, the direction of the load is variable, the safe stress S_s must be taken smaller than when the direction is constant.

5. The values of p for different kinds of journals, according to Unwin, are given in Table I.

For journals not given in Table I, the value of p must be determined by the judgment of the designer. The value adopted should seldom exceed 750 pounds. For journals running faster than about 150 revolutions per minute, p

TABLE II

SAFE WORKING STRESSES ON END JOURNALS

Direction of Load Constant		Direction of Load Variable	
Material	Safe Working Stress S_s	Material	Safe Working Stress S_s
Steel	14,000	Steel	12,000
Wrought iron .	8,500	Wrought iron .	7,000
Cast iron . . .	4,000	Cast iron . . .	3,000

should vary inversely as the number of revolutions. For example, if a given journal is allowed a bearing pressure of 300 pounds at 150 revolutions, it should be allowed a bearing pressure of only $\frac{300 \times 150}{250} = 180$ pounds if it is required to run at 250 revolutions.

6. The permissible working stress S_s may be taken as given in Table II.

NECK JOURNALS

7. A **neck journal** similar to an engine crosshead pin may be considered as a beam supported at both ends and uniformly loaded. Consequently, from *Strength of Materials*, the bending moment is $\frac{w\,l^2}{8} = \frac{P\,l}{8}$.

Then,

$$\frac{P\,l}{8} = \frac{S_s\,\pi\,d^3}{32} \text{ (see formula 1, Art. 2)}$$

and

$$d = \sqrt{\frac{4}{\pi}\frac{P}{S_s} \times \frac{l}{d}} = 1.13\sqrt{\frac{P}{S_s} \times \frac{l}{d}} \qquad (1)$$

Formula **1** may be used to calculate a neck journal when $\frac{l}{d}$ is given or assumed.

But, $P = p\,d\,l$ (see formula **1**, Art. **3**)

Hence,

$$\frac{P\,l}{8} = \frac{p\,d\,l^2}{8} = \frac{S_s\,\pi\,d^3}{32}, \text{ or } \frac{l}{d} = \sqrt{\frac{\pi}{4}\frac{S_s}{p}} = .886\sqrt{\frac{S_s}{p}} \qquad (2)$$

Substituting this value of $\frac{l}{d}$ in formula **1**,

$$d = 1.06\sqrt{\frac{P}{\sqrt{p}\,S_s}} \qquad (3)$$

$$l = \frac{P}{p\,d} \qquad (4)$$

From $P = p\,d\,l$,

The same values for S_s and p may be taken as for end journals.

By comparing formula **3** with formula **3**, Art. **3**, it will be seen that for the same load the neck journal need be only

about two-thirds the diameter of the end journal. A comparison of formula **2** with formula **2**, Art. **3**, shows that the ratio $\frac{l}{d}$ in a neck journal is double what it would be in an end journal.

NOTE.—In using formula **3**, Art. **3**, and formula **3**, Art. **7**, it will be found most convenient to use logarithms. Thus, applying logarithms to formula **3**, Art. **3**, it becomes

$$\log d = \log 1.5 + \tfrac{1}{2}[\log P - \tfrac{1}{2}(\log p + \log S_s)]$$

EXAMPLE.—Find the length and diameter of a wrought-iron neck journal, the load being 9,600 pounds and variable in direction. Allow a bearing pressure of 600 pounds per square inch.

SOLUTION.—Using formula **3**,

$$d = 1.06\sqrt{\frac{9{,}600}{\sqrt{600 \times 7{,}000}}}$$

or, $\log d = \log 1.06 + \frac{1}{2}[\log P - \frac{1}{2}(\log p + \log S_s)]$

$= .02531 + \frac{1}{2}[3.98227 - \frac{1}{2}(2.77815 + 3.84510)] = .36063$

Whence $d = 2.294 = 2\frac{19}{64}$, or $2\frac{5}{16}$, in., nearly. Ans.

$$l = \frac{P}{p\,d} = \frac{9{,}600}{600 \times 2\frac{5}{16}} = 7 \text{ in., nearly. Ans.}$$

8. Sometimes, simply as a matter of taste, it may be desirable to make the length of a journal greater than the calculated value. To give the journal the same strength, its diameter must be increased in the proportion given by the following formula:

$$\frac{l_1}{l} = \left(\frac{d_1}{d}\right)^3$$

where l and d are the original and l_1 and d_1 the new lengths and diameters, respectively, of the journal.

For example, suppose that a wrought-iron end journal is subjected to a load of 350 pounds. Assuming $\frac{l}{d} = 1.4$, the dimensions of the journal will be

$$d = 2.26\sqrt{\frac{P}{S_s} \times \frac{l}{d}} = 2.26\sqrt{\frac{350}{8{,}500} \times 1.4} = .5426 \text{ inch;}$$

$$l = 1.4\,d = 1.4 \times .5426 = .7596 \text{ inch}$$

Hence, call the diameter $\frac{9}{16}$ inch and the length $\frac{3}{4}$ inch.

Though a journal of these dimensions would be sufficiently stiff and durable for the load, it is rather small to

look well. Suppose the length is made double the calculated value; that is, $l_1 = 2l$. Then, from the formula just given,

$$d_1 = d\sqrt[3]{\frac{l_1}{l}} = .5426\sqrt[3]{2} = .6837, \text{ or } 1\tfrac{1}{16}, \text{ inch nearly;}$$

$$l_1 = 2l = 2 \times \tfrac{3}{4} = 1\tfrac{1}{2} \text{ inches}$$

FRICTION OF JOURNALS

9. When relative motion takes place between two surfaces in contact with each other, and subjected to more or less heavy pressure, as in a bearing and a journal, a certain amount of energy is expended in overcoming the frictional resistance to motion. This energy is continually transformed into heat; the temperature of the journal and bearing will therefore increase until a point is reached where the heat is dissipated as rapidly as it is produced. Thus, in designing a journal, the question is not alone that of giving it sufficient strength, but also that of so proportioning it that it may freely radiate the heat developed.

10. The work expended in overcoming the friction of a journal is

$$W = .26\, f P N d \text{ foot-pounds per minute,}$$

where f is the coefficient of friction, and N the revolutions per minute.

It is therefore apparent that with the same load and speed, the work expended against the resistance of friction is directly proportional to the diameter of the journal.

In order to illustrate how the dimensions of a journal affect its heat-radiating qualities, a comparison will be made between two journals of equal lengths, but having different diameters, one being 2 and the other 4 inches. The load P is the same for both journals, but the unit pressure p on the larger journal will necessarily be one-half that on the smaller. The frictional resistance of both journals will be the same, as the total pressures P on the journals are equal; but the amount of work lost in friction and converted into heat will not be the same. If both journals make the same number

of revolutions per minute, the circumferential speed of one is twice that of the other, and as the amount of work performed is the product of the resistance and the distance through which the resistance is overcome, it follows that the energy lost in revolving the larger journal is twice that expended on the smaller one. The temperature attained by both journals will nevertheless be the same, because the 4-inch journal has a radiating surface twice as great as that of the 2-inch one. As should be clearly seen, not only is no advantage derived from an increase in diameter, but, on the contrary, by reason of the enlarged surface, a greater amount of energy is lost in overcoming the frictional resistance. If, on the other hand, the length of the 2-inch journal were doubled, the peripheral speed would remain the same, and likewise the energy lost in friction, but the radiating surface would be doubled.

As another example, take two journals, one 2 inches in diameter and 6 inches long, and the other 4 inches in diameter and 3 inches long. They each have the same projected area; that is, 12 inches. The latter journal, however, requires double the work to overcome friction that the former requires, and, besides, contains twice as much material. Hence, the $2'' \times 6''$ journal is preferable for a steady load. However, the $4'' \times 3''$ journal is $\frac{4^2 \times 6}{2^2 \times 3} = 16$ times as strong as the other, and would be preferred in situations where the load is variable; also, the journal is liable to shocks, as in the case of crankpins of high-speed engines.

11. It must not be inferred from the preceding that an increase in the length of a journal will always overcome its heating tendency, and that in all cases it would be useless to increase the diameter of the journal. A journal running slowly under heavy pressure might heat less by increasing its diameter and thereby reducing the unit bearing pressure. On the other hand, the advantage derived from increasing the length of a journal may be lost by a tendency of the journal to bend, whereby an excess of pressure is produced

at one place, followed by a rise in temperature. By reason of the various points to be taken into consideration, it is difficult to give uniform rules regarding the relation between length and diameter of a journal. In general, the diameter is made as small as the requirements for strength will allow, while the length is made sufficient to keep the bearing pressure within the limits found by experience to be safe.

12. The height of the journal collars, Fig. 1, may be

$$e = \frac{d}{10} + \tfrac{1}{8} \text{ inch}$$

The width of the outer collar is $1\frac{1}{2}\ e$. It is good practice to turn the journal with a fillet in the corner, as the shaft or axle is more liable to crack and fracture if the shoulder is turned with a square corner. The fillet may be outlined by a circular arc drawn with a radius equal to $\frac{1}{2}e$.

PIVOTS AND COLLAR JOURNALS

13. Pivot Journals.—A **pivot journal** is shown in Fig. 3. This type differs from an ordinary journal in that the direction of pressure is parallel with the axis of the shaft instead of perpendicular to it. Thus, the bearing area is the area of the end of the pivot; that is, $.7854\ d^2$.

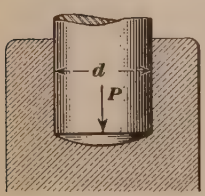


FIG. 3

The diameter of a flat pivot may therefore be found at once by assuming a value for the pressure per square inch of projected area and solving for the area. The formulas in Table III give good results for ordinary cases:

Let P = load on pivot;
d = diameter of pivot;
N = revolutions per minute of pivot.

EXAMPLE.—What should be the diameter of a steel pivot on gun-metal bearings, the load being 200 pounds, and the number of revolutions 320 per minute?

SOLUTION.—

$$d = .004\sqrt{PN} = .004\sqrt{200 \times 320} = 1 \text{ in., nearly.}$$ Ans.

The difficulty met with in pivot journals is the unequal wear of the rubbing surfaces. This is caused by the difference in velocity between parts near the axis of rotation and other parts near the periphery of the journal, the velocity of any particular part being proportional to its distance from the axis of rotation. As the resulting wear of the rubbing surfaces is directly proportional to their respective velocities, it follows that the parts farthest away from the axis of rotation will wear away more quickly. The result is that the

TABLE III

FORMULAS FOR PIVOT JOURNALS

Speed	Wrought-Iron or Steel Pivot on Gun-Metal Bearing	Cast-Iron Pivot on Gun-Metal Bearing	Iron or Steel on Lignum Vitæ Bearing, Moistened With Water	
CASE I—Pivot turning very slowly or intermittently. . .	$d = .035\sqrt{P}$	$d = .05\sqrt{P}$	. . . . .	(1)
CASE II—Revolutions per minute less than 150 . . .	$d = .05\sqrt{P}$	$d = .07\sqrt{P}$	$d = .035\sqrt{P}$	(2)
CASE III—Revolutions per minute more than 150 . .	$d = .004\sqrt{PN}$	. . . . .	$d = .035\sqrt{P}$	(3)

central part of the journal will be gradually exposed to an excess of pressure, followed by abrasion and, possibly, crushing.

14. Collar Journals.—To reduce the extreme differences in velocity between the portions of the bearing surface nearest to and farthest from the centers, it is customary to make use of only the outer part of the rubbing surfaces and to make them ring-shaped. This is the theory on which **collar journals** are based, the aim being to make the collar shallow, thereby avoiding great differences in velocity. In journals exposed to very heavy pressure, the bearing surface is made great enough by increasing the number of collars.

A journal with one collar is shown in Fig. 4, and one with several collars in Fig. 5.

Let d_1 = diameter of collars;
d = diameter of shaft;
b = number of collars;
p = pressure per square inch of projected area;
P = total load, or thrust.

Then, $$.7854\,(d_1^2 - d^2)\,b\,p = P$$

The value of p is usually taken at 60 pounds per square

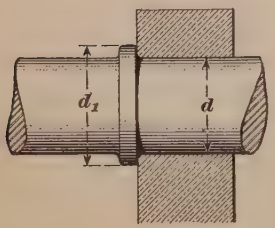


FIG. 4

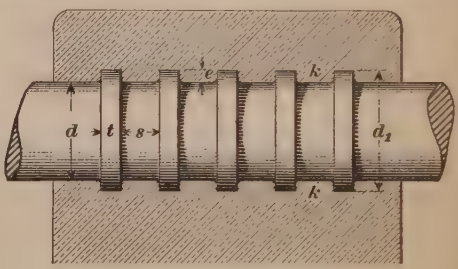


FIG. 5

inch for the thrust bearings of propeller shafts, and this value should not be much exceeded in any case.

Then, $$d_1^2 - d^2 = \frac{P}{47\,b}$$

or, $$d_1 = \sqrt{d^2 + \frac{P}{47\,b}}$$

The number of collars b depends on the judgment of the designer. The larger the number, the smaller is the diameter, and the less will be the wear and work of friction. However, when many collars are used, there is danger of bringing all the thrust on one or two.

In Fig. 5, the dimension $e = \frac{1}{2}(d_1 - d)$. Usually the thickness t of the collars is $.8\,e$, and the width s of the space is equal to the thickness t, unless the annular encircling rings k are hollow for water circulation or unless they are lined with white metal. In the latter case, $s = 2$ to $2\frac{1}{2}\,t$.

EXAMPLES FOR PRACTICE

1. Find the proportions of a cast-iron end journal that turns slowly under a steady load of 15,000 pounds, assuming the length equal to the diameter. Ans. $4\frac{3}{8}$ in. $\times 4\frac{3}{8}$ in.

2. Find the proportions of a wrought-iron end journal that turns under a load of 8,600 pounds, assuming the safe pressure per square inch of projected area to be 650 pounds. The direction of the load is variable. Ans. 3 in. $\times 4\frac{7}{16}$ in.

3. Find the dimensions of a steel neck journal, working under a load of 12,000 pounds, direction variable, the allowable bearing pressure to be 1,200 pounds per square inch. Ans. $1\frac{7}{8}$ in. $\times 5\frac{5}{16}$ in.

4. Find the minimum dimensions of a wrought-iron end journal, the required conditions being that the bearing pressure shall not exceed 750 pounds and that the length shall be one and one-fourth times the diameter. The load on the journal is 9,600 pounds, and variable in direction. Ans. $3\frac{1}{4}$ in. $\times 4$ in.

NOTE.—By using formula **3**, Art. **3**, for solving example 4, the diameter is found to be $3\frac{1}{16}$ inches, nearly. By substituting the value $\frac{l}{1.25}$ for d in formula **4**, Art. **3**, l is found to be equal to 4 inches; but as the quotient $\frac{P}{d\,l}$ in this instance is greater than 750, which is the value of p, the values of d or l must be adjusted to give the desired pressure p. Making $l = 4$ inches, the required value of d may be found by transforming formula **4**, Art. **3**, into the form $d = \frac{P}{l\,p}$, when $d = 3.2$ inches, or $3\frac{1}{4}$ inches, nearly. Ans.

5. Find the diameter of a wrought-iron pivot running in a gun-metal bearing at a speed of 80 revolutions per minute and bearing a load of 800 pounds. Ans. $1\frac{7}{16}$ in.

6. Assuming that five thrust collars on a shaft 10 inches in diameter are subjected to an end thrust of 20,000 pounds, find the diameter and thickness of collars. Ans. $\begin{cases} d_1 = 13.6 \text{ in.} \\ t = 1\frac{7}{16} \text{ in.} \end{cases}$

---

15. Conical Pivot Journals.—When the bearing surface of a pivot journal is conical in shape, as in Fig. 6, it is termed a **conical pivot journal.** Examples of these bearings are found on lathes and similar machines where the end thrust of the journal is not excessive.

The angle u, Fig. 6, of the pivot point does not affect the unit pressure p; this pressure is the same as that of a journal having a flat end whose area is equal to the projection of the conical surface on a plane perpendicular to the axis of

the journal. This can be proved in the following manner: In Fig. 6, the load P may be resolved into the two components S, S acting at right angles to the bearing surface, it being supposed that the load is acting in these two directions only. Then,

$$S = \frac{\frac{P}{2}}{\sin\frac{u}{2}}$$

and

$$2S = \frac{P}{\sin\frac{u}{2}} \qquad (1)$$

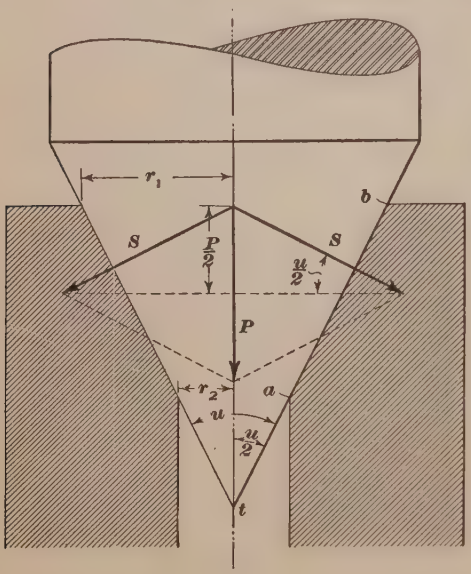


FIG. 6

If A is equal to the surface area of the conical point, then

$$p = \frac{2S}{A}$$

As the active part of the point is the frustum of a cone, the area

$$A = \tfrac{1}{2}(2\pi r_1 + 2\pi r_2)(bt - at)$$

Expressing the lengths bt and at in terms of r_1, r_2, and $\frac{u}{2}$, the formula will be

$$A = \tfrac{1}{2}(2\pi r_1 + 2\pi r_2)\left(\frac{r_1}{\sin\frac{u}{2}} - \frac{r_2}{\sin\frac{u}{2}}\right)$$

or,

$$A = \frac{\pi}{\sin\frac{u}{2}}(r_1^2 - r_2^2) \qquad (2)$$

Inserting this value of A in the formula $p = \frac{2S}{A}$,

$$p = \frac{2S}{\frac{\pi}{\sin\frac{u}{2}}(r_1^2 - r_2^2)}$$

According to formula **1**,

$$2S = \frac{P}{\sin \frac{u}{2}}$$

Therefore,

$$p = \frac{P}{\pi (r_1^2 - r_2^2)}$$

It is seen that the angle u is eliminated from this formula and that it therefore does not affect the unit pressure. The value $\pi (r_1^2 - r_2^2)$ represents the annular surface produced by projecting the conical contact area on a plan perpendicular to the pivot axis.

BEARINGS WITH SLIDING FRICTION

16. Solid Journal Bearings.—The simplest form of bearing for a journal is merely a hole in the frame that supports the rotating piece. Such a bearing is shown in Fig. 7.

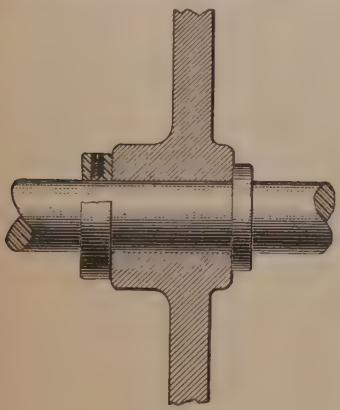


FIG. 7

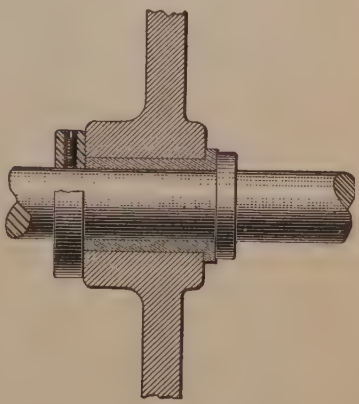


FIG. 8

Motion endwise is prevented by two collars, one of which may be forged on the shaft, and the other made separately and held in place by a setscrew or pin. A boss is cast on the frame, as shown in the figure, in order to give the journal the necessary length and bearing area.

Such a bearing has no means of adjustment to take up the wear; for this reason it is better to use the form of solid

bearing shown in Fig. 8. In this bearing the hole is bored out larger than the journal and is lined with a bushing made of brass or other metal. The wear thus comes on the bushing, which can easily be replaced.

17. Divided Bearings.—In many cases, solid bearings are undesirable, and in others it will be impossible to use them. The bearing is then divided, and the parts held together by bolts; when this is done, the parts of the bushing are called *boxes*, or *brasses*. The division of the bearing permits its adjustment for wear.

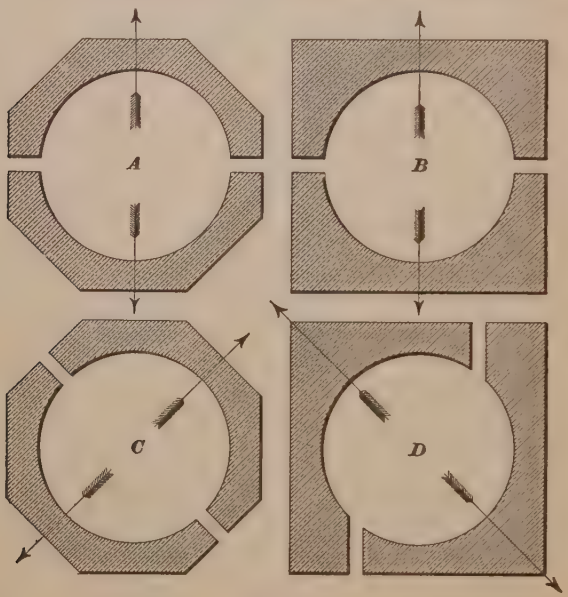


FIG. 9

The load on a bearing usually acts constantly in one direction, and the bearing should be divided so that the line of division is perpendicular, or nearly so, to the direction of the load. Fig. 9 shows various methods of dividing the boxes. At *A* and *B* the direction of the load is vertical; consequently, the boxes are divided horizontally.

Naturally, the wear will come on the top and bottom of the boxes, and the hole will become oval, with the long

diameter vertical. The boxes are then screwed closer together until the hole regains its original circular form. The necessity of making the division perpendicular to the direction of the load is thus apparent. At C the direction of the load is oblique, and the adjustment perpendicular to it; at D the direction of the load is oblique also, but the boxes are divided to allow a horizontal adjustment.

In Fig. 10 is shown a simple form of divided bearing; it is made of cast iron, with no separate boxes, but with means of adjustment. The bearing forms part of the frame of the machine. This type of bearing is used only in cheap work. Sometimes, however, the bearing has a recess, or groove,

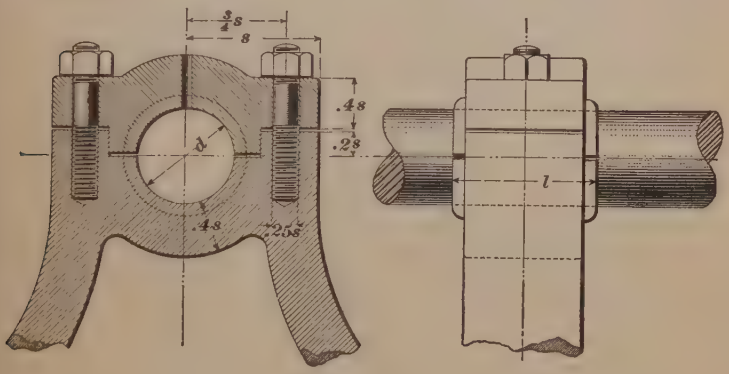


FIG. 10

cast in it, which is filled with Babbitt metal, on which the journal rests. The proportional unit for this bearing is

$$s = 1.15\,d + .4$$

The dimensions l and d of the bearing are the same as those for the journal; the other proportions are given in terms of the unit s, and are shown in the figure.

18. Boxes, or Brasses.—Carefully made bearings are always lined with so-called **boxes**, or **brasses**, which are made of brass, gun metal, phosphor-bronze, white metal, or Babbitt metal, although other alloys are also used. The boxes when worn out may be easily replaced, and being made of softer material than the journal, the latter wears but very little.

An ordinary form of box is shown in Fig. 11. A part of the bearing at each end is made octagonal in cross-section. This part is fitted into an octagonal recess in the *pillow-block*, or *pedestal*, which holds it, and the boxes are thus prevented from turning. Sometimes, the octagonal parts are dispensed with, and the boxes are turned in a lathe, the pillow-blocks, or pedestals, being bored to receive them. It is then necessary to provide the boxes with lugs, or pins, to prevent them from turning.

These boxes may be made according to the following pro-

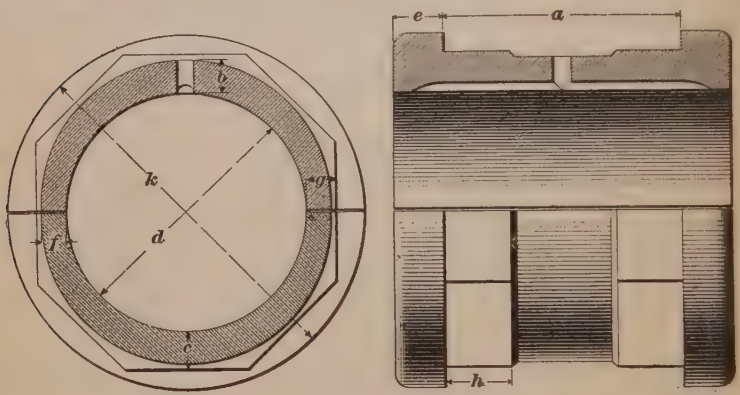


FIG. 11

portions, in which d, the diameter of the journal, is the proportional unit:

$b = \frac{1}{10}\,d + \frac{3}{16}$ in.; $\quad g = \frac{1}{8}\,d$;

$c = \frac{1}{8}\,d + \frac{3}{16}$ in.; $\quad h = \frac{1}{4}\,d$;

$e = \frac{1}{8}\,d + \frac{1}{8}$ in.; $\quad k = 1\frac{1}{2}\,d$;

$f = \frac{1}{10}\,d$; $\quad d =$ diameter of journal

The dimension a should be made to fit the pillow-block for which the boxes are intended.

Boxes should be well supplied with grooves and channels, so that oil may be conducted to every part of the journal. It is a good plan to place a groove parallel with the axis of the journal in the upper half of the bearing, or top box, so as to be in direct communication with the oil holes; then, the advancing side of the journal will always carry a thin film of

oil along with it. The grooves may be from $\frac{1}{8}$ to $\frac{1}{2}$ inch wide, according to the size of the journal.

19. Lining for Boxes.—Boxes for large bearings are often lined with Babbitt, or some other antifriction, metal. Experience has proved that a bearing will run cooler when so lined, probably because the antifriction metal is softer and thus accommodates itself to the journal more readily than the more rigid metal, mostly brass, of which the boxes are made.

Some of the common methods of lining the boxes are shown in Fig. 12. At (*a*), the Babbitt metal is shown cast

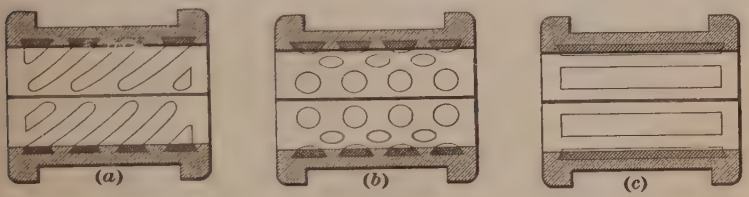


FIG. 12

into shallow helical grooves; at (*b*), into a series of round holes; and at (*c*), into shallow rectangular grooves. Consequently, the journal rests partly on the brass and partly on the Babbitt metal.

In cheap work, the bearing is frequently simply lined with Babbitt metal. A mandrel, the exact size of the journal, is placed inside the bearing, and the melted Babbitt metal is poured around it. In better work, a smaller mandrel is used. After the metal hardens, it is hammered in; the bearing is then bored out to the exact size of the journal.

PEDESTALS

20. The names **pedestal, pillow-block, bearing,** and **journal-box** are applied indiscriminately to one form of bearing, and mean a support for a rotating piece.

21. A form of bearing frequently used for small shafts is shown in Fig. 13. It consists of two parts: the seat

that supports the journal, and the cap that is screwed down to the seat. This bearing is lined with Babbitt metal, or, as it is commonly expressed, the bearing is babbitted. The cap is held in place by capscrews—an invariable method in small pedestals.

22. The proportioning of a pedestal is largely a matter of experience. Few or none of the parts are calculated for

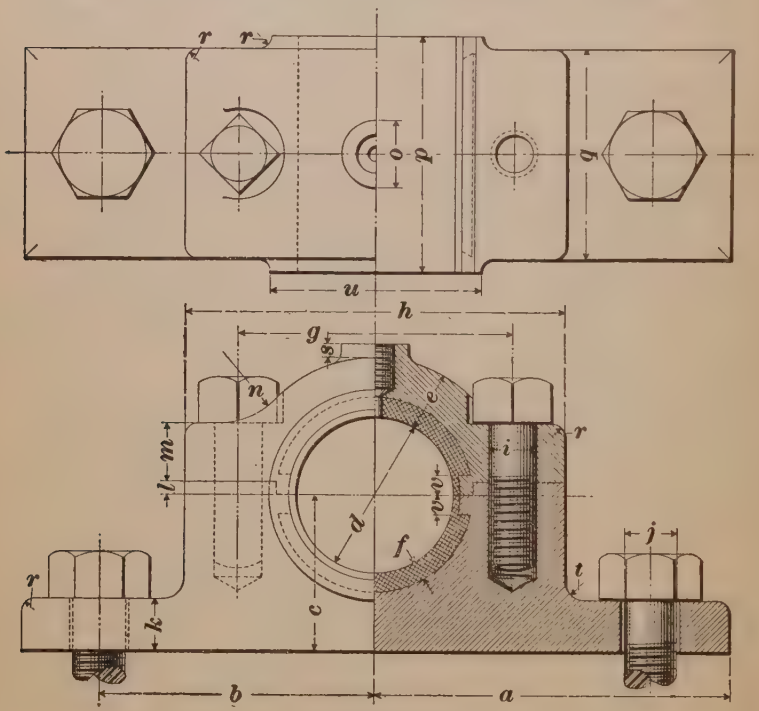


FIG. 13

strength. All the proportions of the pedestals that follow are based on the diameter d of the journal as the unit; the length of the bearing is the same as that of the journal.

For the bearing shown in Fig. 13, the following proportions may be used for sizes of journals from $\frac{1}{2}$ to 2 inches in diameter, inclusive. The diameter d of the shaft is the unit.

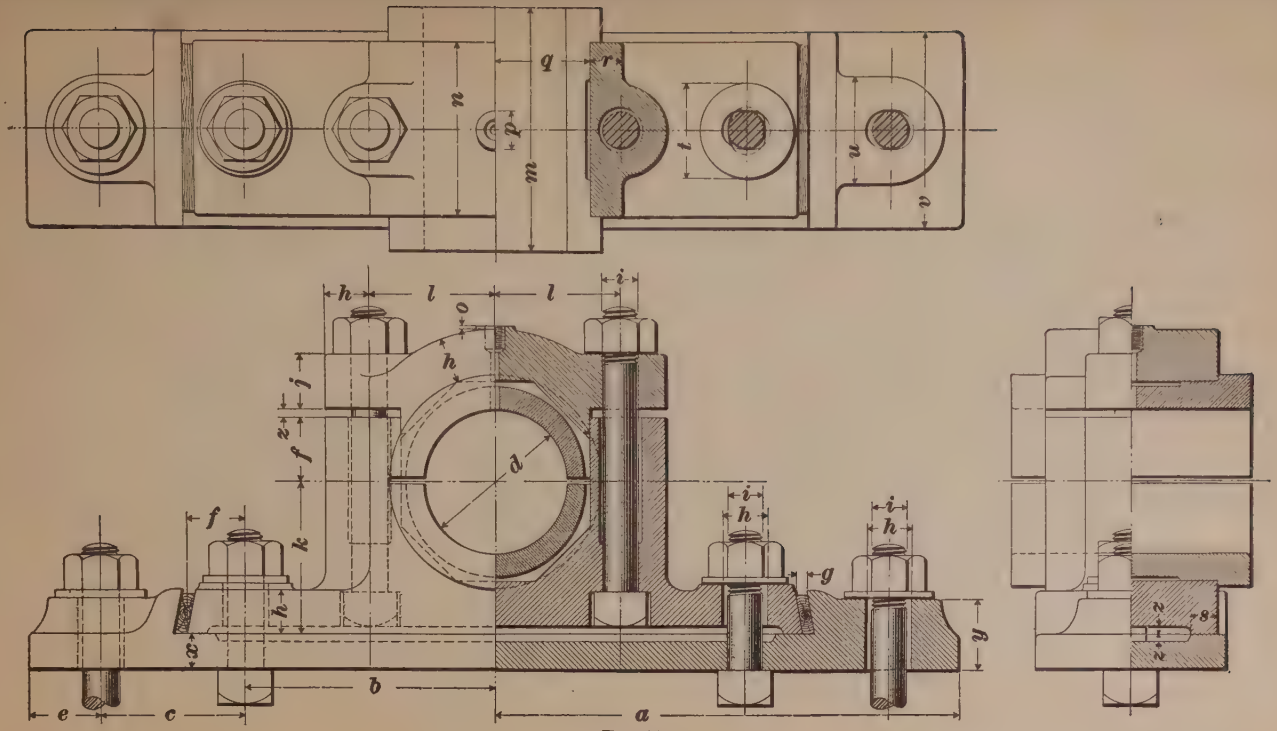


FIG. 14

$a = 2.25\,d$; $l = .08\,d$;
$b = 1.75\,d$; $m = .25\,d + .1875$ in.;
$c = d$; $n = .5\,d$;
$e = .375\,d$; $o = .625$ in. (constant);
$f = .08\,d + .0625$ in.; $p = 1.5\,d$;
$g = 1.75\,d$; $q = 1.333\,d$;
$h = 2.45\,d$; $r = .08\,d$;
$i = .3\,d$; $s = .125$ in. (constant);
$j = .33\,d$; $t = .16\,d$;
$k = .25\,d + .125$ in.; $u = 1.333\,d$;
$v = .125\,d$.

23. In Fig. 14 is shown a common form of pedestal used for somewhat larger journals than the one shown in Fig. 13. This pedestal consists of: (1) a foundation plate that is bolted to the foundation on which the pedestal rests; the plate is essential when the pedestal rests on the brickwork or masonry, but may be dispensed with when the pedestal rests on the frame of the machine; (2) the block that carries the brasses and supports the journal; and (3) the cap that is screwed down over the brasses. The bolt holes in both the foundation plate and the block are oblong, so that the pedestal may be readily adjusted.

The following proportions may be used for this kind of pedestal, with journals from 2 to 6 inches in diameter, inclusive. An oil cup having a $\frac{1}{4}$-inch pipe tap shank may be used on pedestals for journals having diameters from 3 to 4 inches, and $\frac{3}{8}$-inch pipe tap shank for larger sizes up to 6 inches in diameter. The diameter d of the journal is the unit.

NOTE.—The shanks of oil cups and grease cups in the market are made with $\frac{1}{8}$-, $\frac{1}{4}$-, $\frac{3}{8}$-, and $\frac{1}{2}$-inch pipe threads. The amount of oil or grease the cup holds when filled is usually expressed in ounces.

$a = 3.25\,d$; $g = .09\,d$;
$b = 1.75\,d$; $h = .3125\,d$;
$c = d$; $i = .25\,d$;
$e = .5\,d$; $j = .375\,d$;
$f = .4375\,d$; $k = 1.0625\,d$;

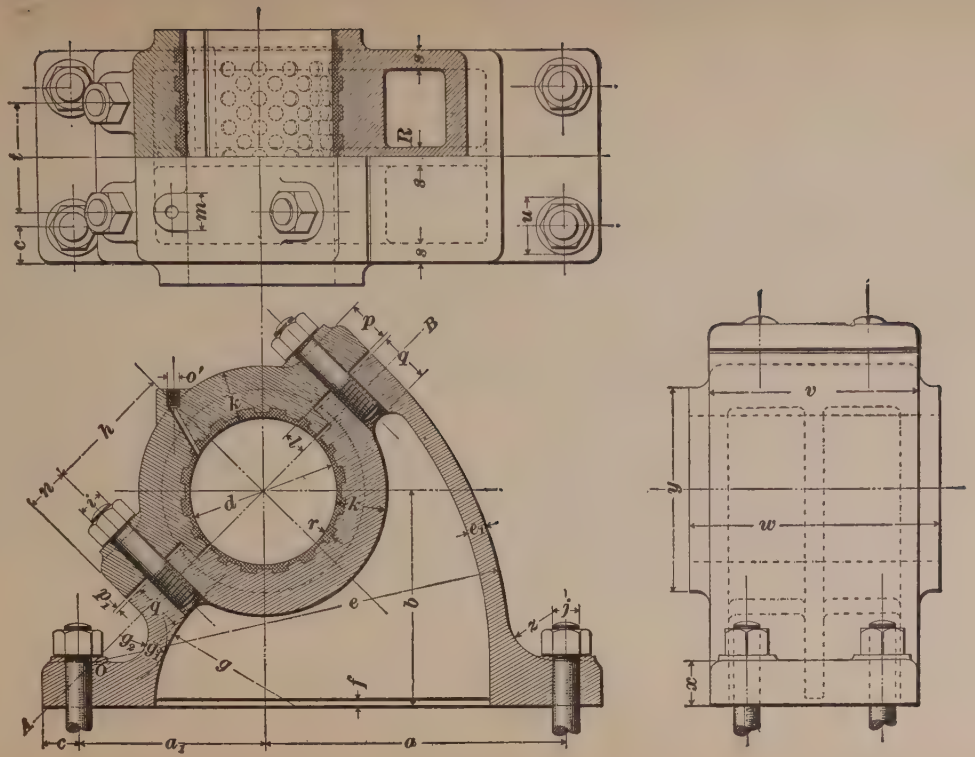


FIG. 15

$l = .875\,d$;
$m = 1.75\,d$;
$n = 1.25\,d$;
$o = .125$ in. (constant);
$p = .875$ in. (constant);
$q = .625\,d$;
$r = .25\,d$;
$s = .1875\,d$;
$t = .65\,d$;
$u = .75\,d$;
$v = 1.375\,d$;
$x = .25\,d$;
$y = .5\,d$;
$z = .0625\,d$.

24. Crank-Shaft Pedestals.—The load on a crank-shaft bearing is due partly to the weight of the shaft and flywheel, and partly to the alternate push and pull of the connecting-rod. The direction of the resultant of these two forces is therefore more or less oblique; consequently, the pedestal is often divided obliquely.

25. A pedestal of the kind just mentioned is shown in Fig. 15, and may be proportioned by taking the diameter d of the journal as the unit and using the following proportions:

$a = 2\,d + 1$ in.;
$a_1 = 1.2\,d + 1$ in.;
$b = 1.5\,d$;
$c = .22\,d + .5$ in.;
$e_1 = .1\,d + .25$ in.;
$f = .5$ in. (constant);
$g_1 = .1\,d + .25$ in.;
$h = .9\,d$;
$i = .15\,d + .25$ in.;
$j = .15\,d + .375$ in.;
$k = .3\,d + .625$ in.;
$l = .1\,d + .375$ in.;
$m = .25\,d$;
$n = .3\,d$;
$p = .25\,d + .25$ in.;
$p_1 = .04\,d$;
$q = .35\,d$;
$r = .02\,d + .3125$ in.;
$s = .1\,d + .25$ in.;
$t = .75\,d$;
$u = .27\,d + 1.125$ in.;
$v = 1.45\,d$;
$w = 1.75\,d$;
$x = .25\,d + .625$ in.;
$y = 1.3\,d + 1$ in.;
$z = .25\,d + 1$ in.

The line $A\,B$ is at an angle of 45° to the base line. This pedestal, as shown, is babbitted. The Babbitt is held in place and prevented from turning with the journal by providing the surfaces with which the Babbitt comes in contact with round projections, as shown. The projections are about 1 inch in diameter for the larger sizes of pedestals.

To find the radius e, draw a line parallel with, and at a distance x above, the base line. The point of intersection O

of this line with the line AB is the center of the arc having the radius e. The radius g_2 is found by trial. The center of the arc must lie on the line AB. With a radius $g_2 + g_1$, describe an arc from the same center. Draw a line parallel

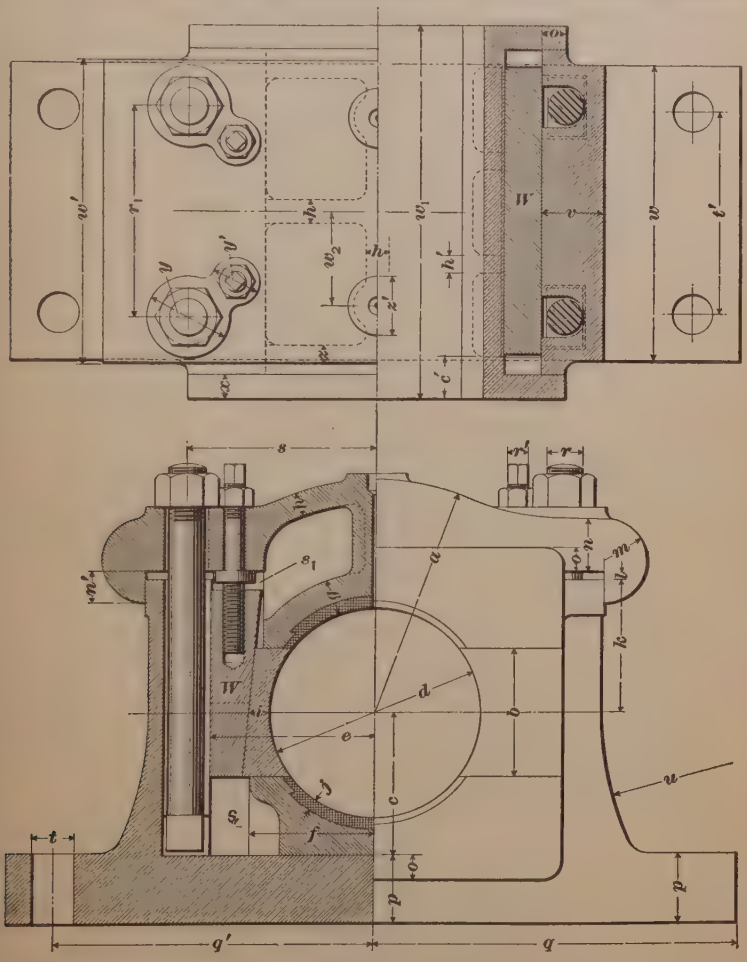


FIG. 16

with the face of the bearing at a distance from it determined by q. The radius g is to be found by trial, the center being on the base line, and the arc tangent to the line determined

by q and to the arc determined by $g_2 + g_1$. The rib R may be used on all pedestals in which the diameter of the journal is above 6 inches. For 3- to 4-inch journals, o' may be tapped with a $\frac{1}{4}$-inch pipe tap; up to 6-inch journals, a $\frac{3}{8}$-inch pipe tap; up to 9-inch, a $\frac{1}{2}$-inch pipe tap; and all sizes above 9 inches, a $\frac{3}{4}$-inch pipe tap.

Two oil cups may be used on all bearings in which the diameter of the journal is above 8 inches. This style of pedestal may be used for journals from 3 inches up to 15 inches in diameter, inclusive.

26. Fig. 16 shows a pedestal suitable for the crank-shaft of a horizontal engine with journals from 8 to 20 inches in diameter. The block may be complete in itself, as shown in the figure, but more often it forms part of the engine bed.

The box is in three parts, the cap, the cheek boxes, or cheeks, and the bottom box. The cheeks may be adjusted horizontally by means of wedges W, and the bottom box may be raised by placing packing pieces under it. To obtain the dimensions of this pedestal, use the following proportions, which are based on the unit d, or the diameter of the crank-shaft journal:

$a = d + 1$ in.;
$b = .5\,d + 1$ in.;
$c = .66\,d$;
$c' = .2\,d$;
$e = .825\,d - .25$ in.;
$f = .6\,d$;
$g = .1\,d + .5625$ in.;
$h = .1\,d + .25$ in.;
$h' = .08\,d$;
$i = .11\,d$;
$j = .625$ in. (constant);
$k = .5\,d + 1.25$ in.;
$l = .375$ in. (constant);
$m = .175\,d + .3125$ in.;
$n = .25\,d + .25$ in.;
$n' = .1\,d + .375$ in.;
$o = 1$ in. (constant);
$p = .25\,d + .625$ in.;
$q = 1.75\,d$;
$q' = 1.5\,d$;
$r = .15\,d$;
$r' = .1\,d$;
$r_1 = d$;
$s = .9\,d$;
$t = .15\,d + .375$ in.;
$t' = .9\,d$;
$u = 1.5\,d$;
$v = .25\,d + .375$ in.;
$w = 1.45\,d$;
$w' = 1.47\,d$;
$w_1 = 1.75\,d$;
$w_2 = \frac{w_1}{4}$;
$w_3 = w_1 - 2\,x$;

$w_4 = w_1 - 2\,c' + \frac{1}{8}$ in.; $y' = .2\,d + .5$ in.;
$x = .1\,d$; $z = .09\,d$;
$y = .3\,d + .75$ in.; $z' = 2.5$ in. (constant).
Taper of adjusting wedge $= 1\frac{1}{4}$ inches per foot.

The form and location of the wedge W are shown in the plan view, Fig. 16, and the perspective views, Fig. 17 (b) and (c). It will be noticed that the upper part of the wedge extends across the whole width of the pedestal, until it reaches the flanges of the cap and the cheeks which have corresponding recesses s_1, s_1, Fig. 17 (a) and (b). As shown

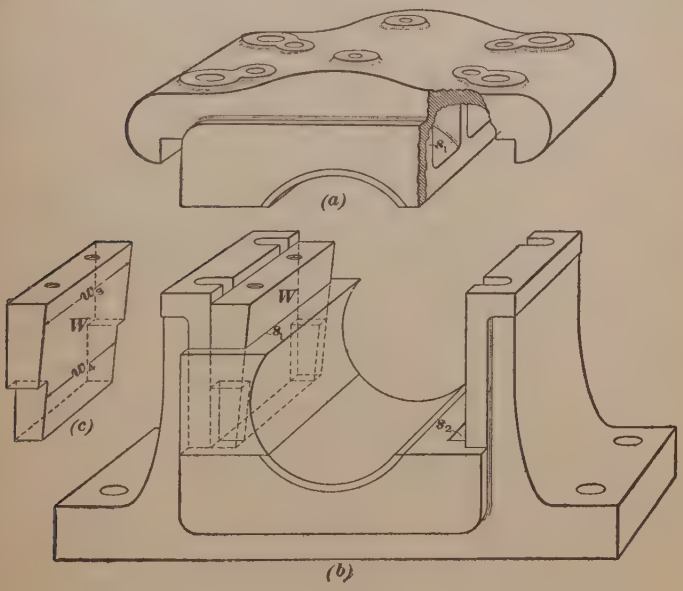


FIG. 17

in Fig. 17 (b), the bottom box has shoulders that make the recess s_2 narrower than those in the cheeks and cap. For this reason, the lower part of the wedge is made narrower, so as to enter the recess s_2. The perspective views will assist in locating the various parts and in understanding the peculiar shape of the cap, the view of the latter being partly broken away in order to show the lower central part.

The dimensions of the wedge may be found in the following manner: Lay off the lengths i and e, Fig. 16, along the horizontal center line of the bearing, the difference between the two dimensions being equal to the thickness of the wedge along this line. Through the left-hand end of dimension i draw a vertical line equal in length to b, the lower end of the line beginning at the lower side of the cheek. As the taper of the wedge is $1\frac{1}{4}$ inches to the foot, calculate the amount of taper for a length equal to b. Draw horizontal lines through the upper and lower ends of this vertical line, and along the upper line set off one-half of the taper to the right of the vertical line, and along the lower one, one-half to the left. By connecting these two points, the taper side of the wedge is determined; this side may now be extended upwards to any point desired, the bottom of the wedge being made flush with the lower side of the cheek.

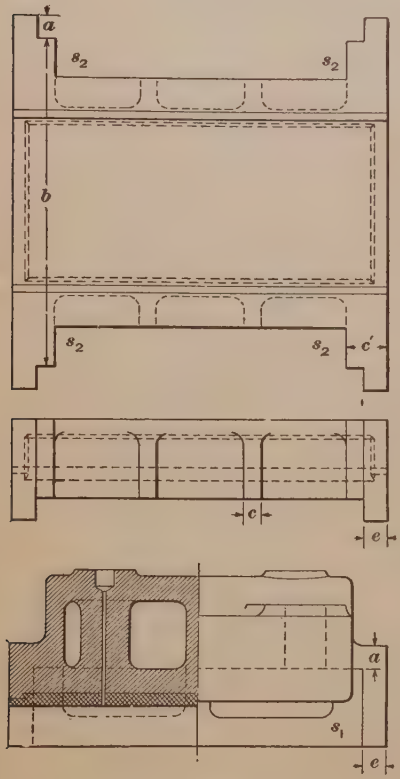


FIG. 18

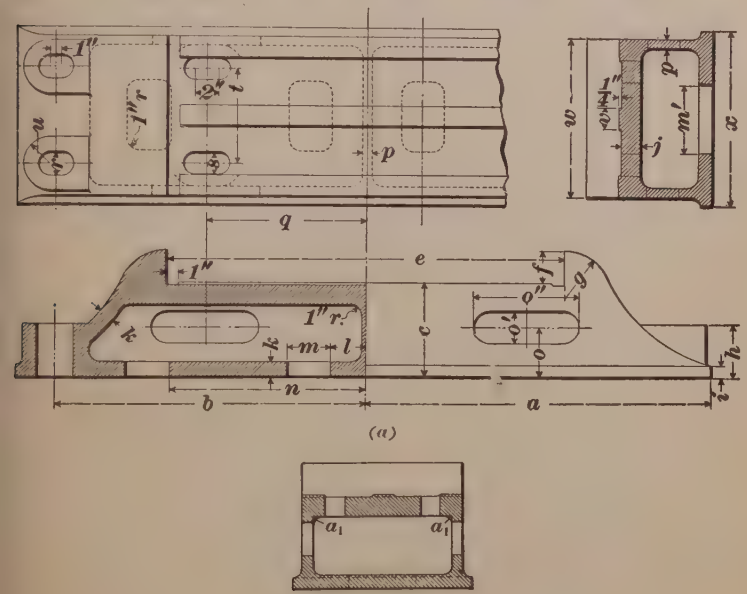
The center of the circle of diameter y', shown in the plan view, Fig. 16, should be approximately over the center of the adjusting wedge and far enough from the center of the large bolt to permit both nuts to be turned freely.

Further details of the bottom box and the cap are shown in Fig. 18, in which the unit is the same as in Fig. 16, and the proportions are as follows:

$a = 1$ in. (constant); $c' = .2\,d$;
$b = 1.65\,d - .5$ in.; $e = .1\,d$.
$c = .08\,d$;

The outlines of the dovetailed projections that hold the Babbitt lining in place are indicated by dotted lines.

27. The foundation, or bed casting, is shown in Fig. 19 (*a*), and has dimensions to suit the pedestal shown in Fig. 16. The



(*a*)

(*b*)

FIG. 19

proportions for this foundation are as follows, the diameter d of the crank-shaft journal being taken as the unit.

$a = 2.45\,d + 7.25$ in.; $i = .05\,d + .5$ in.;
$b = 2.3\,d + 5.25$ in.; $j = .05\,d + 1.125$ in.;
$c = .5\,d + 3.5$ in.; $k = .05\,d + .75$ in.;
$e = 3.5\,d + 2$ in.; $l = .25\,d + .75$ in.;
$f = .25\,d + .5$ in.; $m = .4\,d$;
$g = .25\,d + 1.75$ in.; $m' = .6\,d$;
$h = .25\,d + 2.25$ in.; $n = 1.55\,d + 2.5$ in.;

$o = .25\,d + 2$ in.; $s = .15\,d + .375$ in.;
$o' = .25\,d + .5$ in.; $t = .9\,d$;
$o'' = .5\,d + 4.5$ in.; $u = .15\,d + .875$ in.;
$p = .08\,d$; $v = .2\,d$;
$q = 1.5\,d$; $w = 1.5\,d$;
$r = .15\,d + .375$ in.; $x = 1.65\,d$.

Fig. 19 (*b*) shows a cross-section taken through two of the holes for the pedestal bolts, and explains the presence

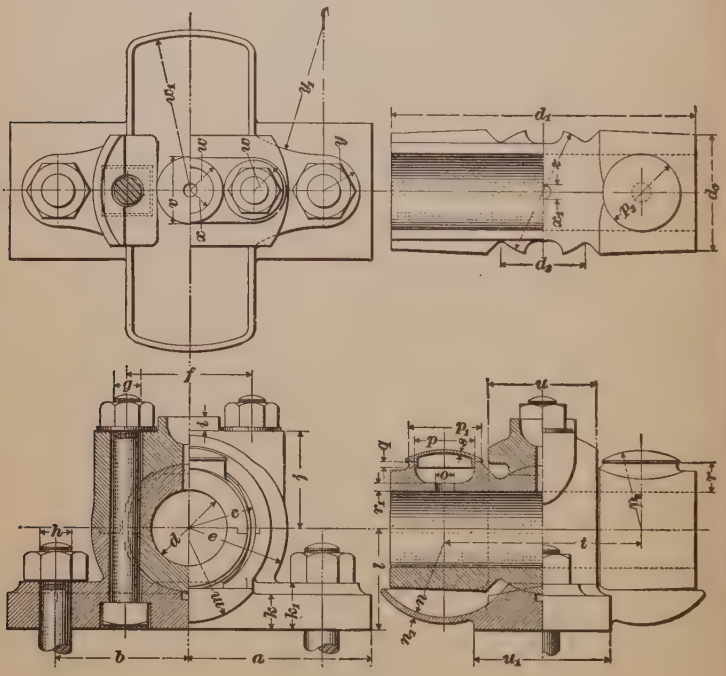


FIG. 20

of the dotted curve at the outside of the holes in the plan view. As shown at a_1, the fillet at the inside of the top has been milled away near the bolt hole, in order to make room for the pedestal bolt head and give the latter a flat bearing surface.

28. Ball-and-Socket Bearings.—The **ball-and-socket bearing** now largely used in the United States was

first introduced by William Sellers & Co., of Philadelphia. This bearing has very long boxes made of cast iron bored to fit the journal. In some cases, however, the boxes are cast with a recess in which a Babbitt lining is poured to form a bearing surface for the journal. The boxes have a spherical enlargement at the center, which fits into corresponding hollows in the block and cap, thus making a ball-and-socket joint, which leaves the box free to move slightly in any direction to conform to an inequality or want of alinement in the shaft. When rigid boxes are used, they must necessarily be short on account of unavoidable deflections of the shaft due to belt pull, thrust of gearing, etc. Pivoted boxes, on the contrary, by reason of their flexibility, may be made long, thereby giving a large wearing surface and increased durability.

29. Fig. 20 shows a pedestal with ball-and-socket bearing. The boxes are made of cast iron and have a length equal to four times the diameter of the journal. Ordinarily, the journal is lubricated through the oil hole in the center of the cap, but the top box also has two cups, which are filled with a mixture of tallow and oil that melts at about 100° F. If the bearing becomes heated, this mixture melts, and thus helps in the lubrication.

The dimensions of this bearing are obtained from the following proportions, which are based on the diameter d of the journal as the unit:

$a = 2.375\,d$;
$b = 1.75\,d$;
$c = .875\,d$;
$d_1 = 4\,d$;
$d_2 = 1.1\,d$;
$d_3 = 1.25\,d + .5$ in.;
$e = 1.3\,d$;
$f = 1.65\,d$;
$g = .2\,d + .25$ in.;
$h = .2\,d + .375$ in.;
$i = .1875\,d$;
$j = 1.27\,d$;
$k = .3\,d + .25$ in.;
$k_1 = .4\,d + .375$ in.;
$l = 1.33\,d$;
$m = d + .125$ in.;
$n = d$;
$n_1 = .125$ in. (constant);
$o = .25\,d$;
$p = .8\,d$;
$p_1 = .95\,d$;
$p_2 = d$;

$p_2 = d$;
$q = .07\,d$;
$r = .4\,d$;
$r_1 = .1\,d$;
$s = .0625$ in. (constant);
$t = 2.5626\,d$;
$u = 1.45\,d$;
$u_1 = 1.8\,d$;
$v = .875\,d$;
$w = .2\,d + .375$ in.;
$w_1 = 2.06\,d$;
$x = .18\,d + 2.5$ in.;
$x_1 = .1875\,d$;
$y = .25\,d + .375$ in.;
y_1 to be found by trial;
$z = 1.5\,d + .3125$ in.

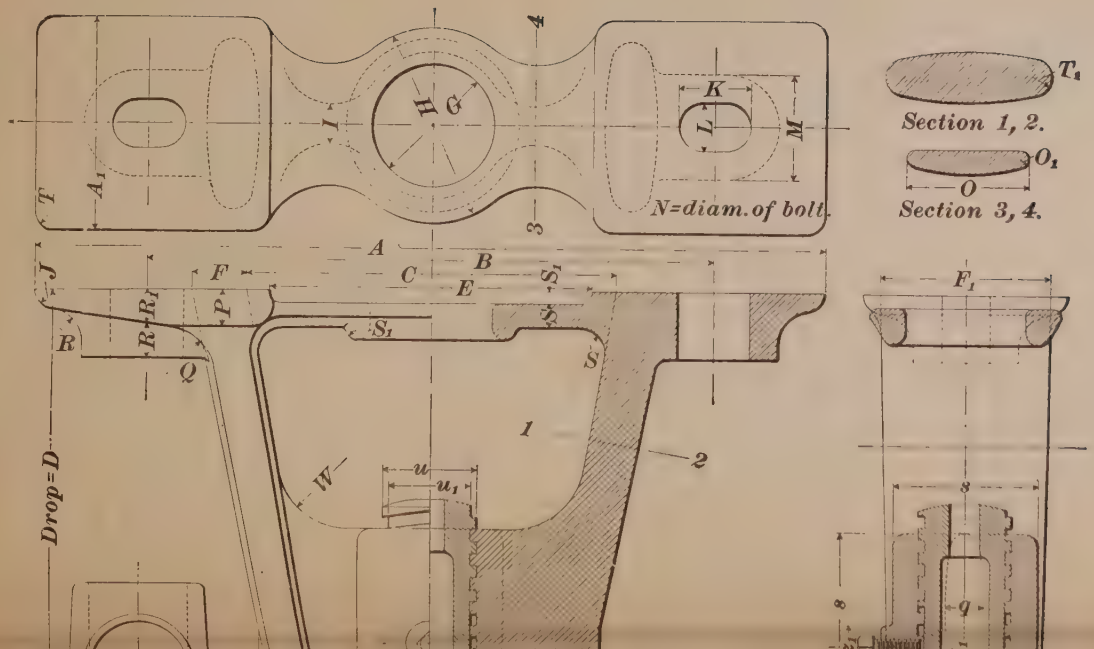
WALL BRACKETS AND HANGERS

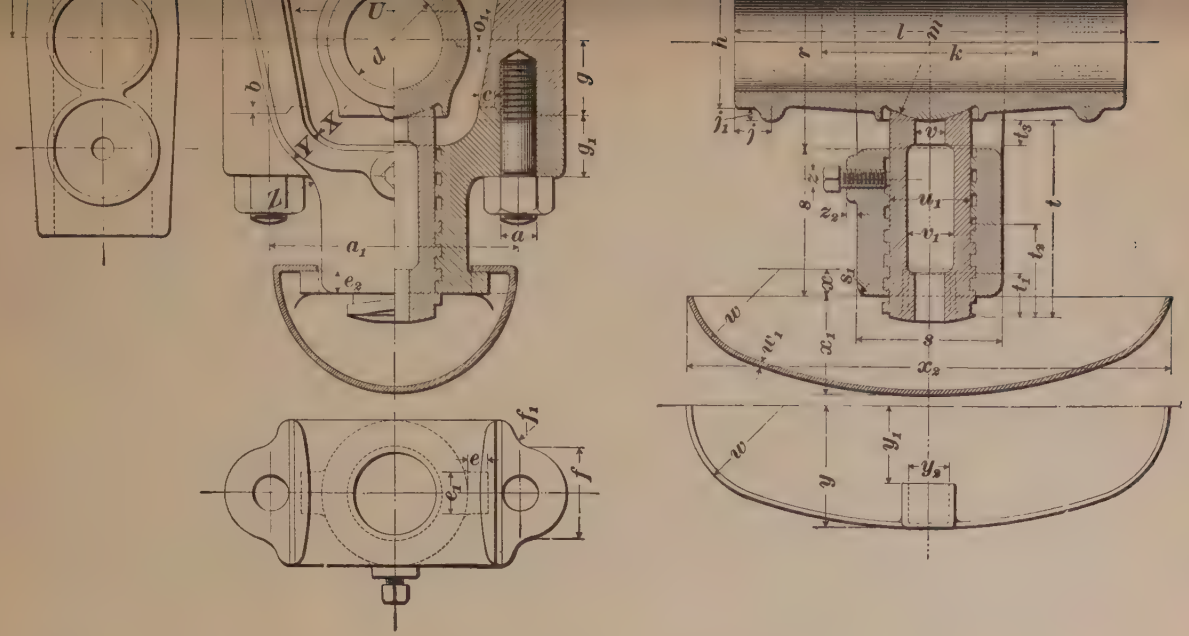
30. Wall Brackets.—A shaft must sometimes be supported by a bearing fixed to a wall or pillar. In such a case, the bearing is generally supported by a **wall bracket.** It will readily be seen that the bracket is a cantilever with practically a uniform load, due to its own weight, and a concentrated load, due to the weight of the shaft; hence, the bending moment may be determined by the principles of strength of materials.

In order that a cantilever shall be equally strong at all sections, the resisting moment $\frac{SI}{c}$ of that section must be proportional to the bending moment at that section. Suppose the bracket in question to be a plate of a constant thickness b and a varying height h. Then, at any section, the bending moment

$$M = \frac{SI}{c} = \frac{S\frac{1}{12}bh^3}{\frac{1}{2}h} = \frac{1}{6}Sbh^2$$

Since $\frac{1}{6}Sb$ remains constant for all sections, h^2 is proportional to the bending moment; or, in other words, the height of a cantilever at any section should be proportional to the square root of the bending moment at that section. It can be shown mathematically that a cantilever of constant width should have a parabolic form when the load is concentrated at the end, and a triangular form when the load is uniformly distributed. In practice, a bracket or other machine part of the cantilever class is given a form that is often neither exactly parabolic nor triangular, but approximates closely


T
A1
I
H
G
K
L
M
N=diam. of bolt.
4
3
T1
Section 1, 2.
O1
O
Section 3, 4.
J
A
C
B
E
F
S1
P
R1
R
Q
S
Z
1
2
W
u
u1
Drop=D
F1
8
q



BB 225 997C

Fig. 21

to one or the other, according to the taste and judgment of the designer. The bearing of the wall bracket may be made the same as in the hanger shown in Fig. 21.

31. Hangers.—A **hanger** is used when a shaft bearing is to be suspended from the ceiling. Fig. 21 shows a form of a hanger frequently employed.

The frame of the hanger is divided, and the parts are connected by bolts. With such a form the shaft may be more easily removed than when the hanger frame is a solid piece.

The units for determining the leading dimensions of a shaft hanger are the diameter d of the shaft and the drop D of the hanger. The following proportions are suitable for shafts ranging from $1\frac{1}{2}$ inches up to $4\frac{1}{2}$ inches in diameter:

$A = 6\,d + .45\,D$;
$A_1 = 2\,d + .03\,D$;
$B = 4\,d + .35\,D$;
$C = 2\,d + .3\,D$;
$E = 2\,d + .25\,D$;
$F = .5\,d + .01\,D$;
$F_1 = 1.5\,d + .05\,D$;
$G = 1.25\,d$;
$H = 2\,d$;
$I = .4\,d$;
$J = .125\,d + .01\,D$;
$K = .5\,d + .5$ in.;
$L = .25\,d + .5$ in.;
$M = .75\,d + .6875$ in.;
$N = .25\,d + .375$ in.;
$O = 1.25\,d$;
$O_1 = .094\,d + .002\,D$;
$P = .375\,d + .008\,D$;
$Q = .375\,d + .008\,D$;
R and R_1 = (see note);
$S = .25\,d + .005\,D$;
$S_1 = .125\,d + .003\,D$;
$T = .125\,d + .01\,D$;
T_1 = (see note);
$U = 2\,d$;
$V = .5\,d$;
$W = .75\,d$;
$X = .375\,d$;
$Y = .25\,d + .125$ in.;
$Z = .625\,d$;
$a = .15\,d + .375$ in.;
$a_1 = 2.4\,d + .3125$ in.;
$b = .08\,d$;
$c = .125\,d + .0625$ in.;
$e = .2\,d$;
$e_1 = .4\,d$;
$e_2 = .2\,d$;
$f = .375\,d + 1$ in.;
$f_1 = .09\,d + .25$ in.;
$g = .75\,d$;
$g_1 = 1.3125\,d + .125$ in.;
$h = 1.25\,d + .1875$ in.;
$i = .1\,d$;
$j = .25\,d + .25$ in.;
$j_1 = .125\,d + .0625$ in.;
$k = 2.2\,d$;
$l = 4\,d$;
$m = 1.4\,d + .375$ in.;

$n = d$;
$o = .25\,d$;
$o_1 = .0625\,d$;
$p = d$;
$p_1 = .0625\,d$;
$q = .4\,d$;
$q_1 = .15\,d$;
$r = 2.125\,d$;
$s = 1.5\,d$;
$s_1 = .125\,d$;
$t = 2\,d$;
$t_1 = .5\,d$;
$t_2 = d$;
$t_3 = .25\,d$;
$u = .95\,d$;
$u_1 = .85\,d$;
$v = .25\,d + .125$ in.;
$v_1 = .5\,d$;
$w = d$;
$w_1 = .125$ in. (constant);
$x = .25\,d$;
$x_1 = d$;
$x_2 = 4\,d + 2$ in.;
$y = 1.25\,d$;
$y_1 = .75\,d + .0625$ in.,
$y_2 = .4\,d + .0625$ in.;
$z = .06\,d + .25$ in.;
$z_1 = .12\,d + .75$ in.;
$z_2 = .3125$ in. (constant).

Thread of plugs = .5-inch pitch for all sizes.

NOTE.—To find R_1, draw the arc J; also, draw the arc Q tangent to P; then, draw a straight line tangent to these arcs, and R_1 will be the distance along the center line, determined by B included between this tangent and the upper face of the hanger. Having found R_1, make R equal to it.

The radius T_1 is made equal to three-eighths the thickness at the middle.

The boxes of the ball-and-socket bearings shown in Figs. 20 and 21 are made of cast iron, and are bored to fit the journal without lining. The ball, and the recesses in the ends of the plugs into which the ball is fitted, should be finished. The screw threads on the plugs, Fig. 21, may be either cast on the plugs or turned, the latter method being preferable.

The spherical enlargement at the center of the boxes is held between the ends of two hollow cast-iron plugs, which are threaded at their outer ends and screw into bosses cast on the bracket. The boxes may be raised or lowered by screwing the plugs up or down. In order to turn the plugs, the hole at the end is made square to receive a key. When the plugs are in the desired position they are locked by setscrews.

32. Pivot, or Footstep, Bearings.—Bearings that are used to support the ends of vertical shafts are known as **pivot,** or **footstep, bearings.** An ordinary pivot bearing

is shown in Fig. 22. The end of the pivot rotates on a disk A, which may be made of steel, brass, or bronze. The brass bushing B prevents the pivot from moving sidewise. The end of the pivot should be made of steel, and it may be flat on the end or slightly convex. The proportions are

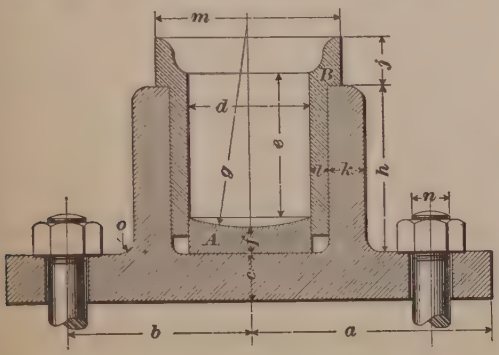


FIG. 22

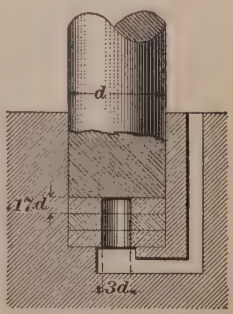


FIG. 23

given in terms of the diameter d of the pivot as a unit, and are as follows:

$a = 2\,d$;
$b = 1.5\,d$;
$c = .25\,d + .375$ in.;
$e = 1.25\,d$;
$f = .2\,d + .125$ in.;
$g = 1.75\,d$;
$h = 1.4\,d$;
$j = .4\,d$;
$k = .3\,d$;
$l = .15\,d$;
$m = 1.5\,d$;
$n = .2\,d + .25$ in.;
$o = .15\,d$.

33. Pivots With Loose Disks.—When pivots are required to run at great speed, the relative speed of the surfaces in contact may be reduced by placing a number of loose disks between the pivot and the footstep, shown in Fig. 23. These disks are sometimes made slightly convex.

If a is the number of disks and n the revolutions per minute of the pivot, then the relative speed of any of the surfaces in contact is $\frac{n}{a+1}$ revolutions per minute. Suppose, for example, a pivot runs at 3,000 revolutions per minute, and it is desired to reduce the speed between adjacent surfaces to

750 revolutions per minute; then, $750 = \frac{3{,}000}{a+1}$, or $a = 3$, the number of disks required. The pivot and disks may be lubricated by a channel, as shown in the figure. The disks may be made of gun metal or phosphor-bronze. Proportions are given in the figure.

ROLLER AND BALL BEARINGS

ROLLER BEARINGS

34. Rolling Friction.—In Fig. 24, C illustrates a cylinder at the center of which the force P is acting, compelling the cylinder to roll over the surface D and to revolve in the direction shown by the arrow v. As indicated, the surface D undergoes a certain amount of deformation, which may be permanent, if the material is of a plastic nature, or only temporary, if the material is elastic and the compressive stress does not exceed the elastic limit. In the position shown, the point e is lower than b, and in order that the motion may continue, the cylinder must revolve about b as an instantaneous axis of rotation. The moment causing rotation is $P \times ab$, and the resisting moment is $W \times bc$, W being the load supported by the cylinder. These moments must be equal; hence,

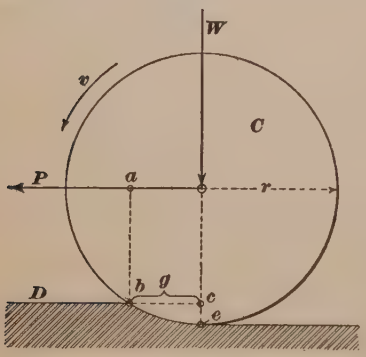


FIG. 24

$$P \times ab = W \times bc, \text{ and } P = \frac{W \times bc}{ab}$$

If $bc = g$ and the cylinder radius is r, then

$$P = \frac{Wg}{r - ce}$$

For hard materials, as used in bearings, the length ce is very small compared with r, and may be left out of consideration. The formula then becomes

$$P = \frac{Wg}{r}, \text{ approximately}$$

This formula shows that the rolling resistance decreases with an increase of r.

Very few experiments have been made to determine the value of g. Some tests give g a value of from .007 to .015 inch for steel wheels rolling on steel rails.

As the rolling resistance between two polished metal surfaces is very low, the advantage to be derived from this combination has been utilized in the construction of roller and ball bearings. In all bearings of this class, it is important that the rollers or balls, as well as the paths provided for them, be of the greatest hardness and smoothness. For this reason, if the whole of the parts in contact cannot be made of the required hardness, it is customary to provide them with linings that possess the requisite qualities.

35. Cylindrical Rollers.—The principal parts of a roller bearing, with the accessory parts omitted, are shown in Fig. 25, (a) being a longitudinal section and (b) an end

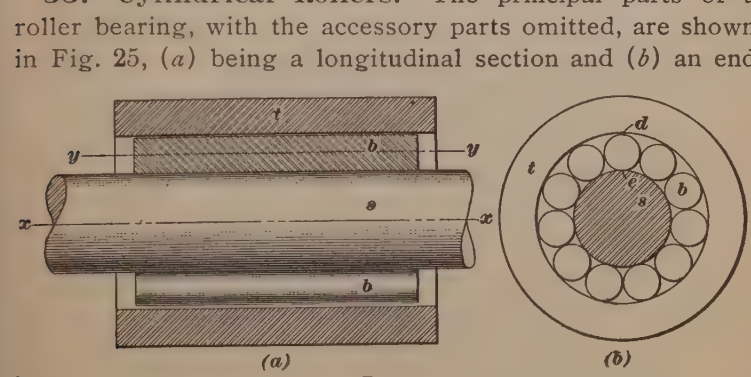


FIG. 25

view. The part s may represent part of a shaft or journal surrounded by a series of rollers b, b, which are held in position by means of the box t. In order that pure rolling may take place, the casing, rollers, and journal must be perfectly cylindrical; also, the axes $y\,y$ of the rollers and the axis $x\,x$

of the journal must be parallel, and endwise motion of the rollers must be prevented. For the purpose of holding the rollers parallel to one another and to the journal, various methods are in use, as slotted cylinders, or cages, rings with pins, and rods guiding and separating the individual rollers.

Rollers may be conical as well as cylindrical. In order that true rolling may be insured with conical rollers, it is necessary that all the axes of rotation of the rollers shall intersect the axis of the journal at the same point, the position of which remains constant throughout a revolution of the journal.

The application of this rule may be more clearly shown by means of Fig. 26, where the rollers b, as well as the journal s, are conical. The axes $y\,y$ of the rollers, produced, intersect

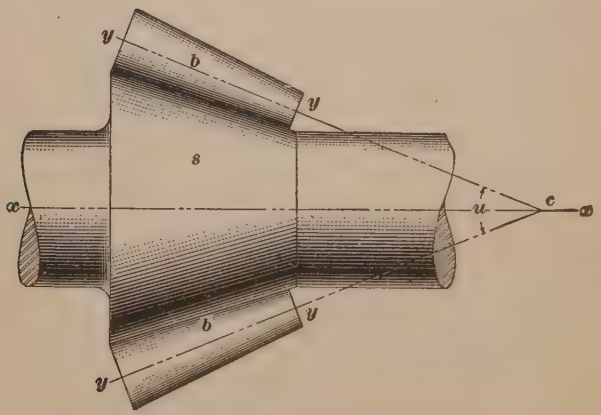


FIG. 26

the axis $x\,x$ at a point c. If the position of this point is unaltered while the journal completes a revolution, true rolling takes place.

36. According to the preceding rule, there are cases where true rolling should occur, but where, nevertheless, sliding friction is present. An instance of this kind is shown in Fig. 27, which may be considered as a variation of Fig. 26, the angle u being increased to 180° so that the axes $y_1 y_1$ and $y_2 y_2$ are at right angles to $x\,x$. In Fig. 27, b_1 and b_2 are

cylindrical rollers resting on the bearing plate t and supporting the circular end of the shaft s. If the rollers are guided by suitable means, such as a perforated disk, the intersection of the axes $y_1 y_1$ and $y_2 y_2$ with the axis $x\,x$ of the shaft will be

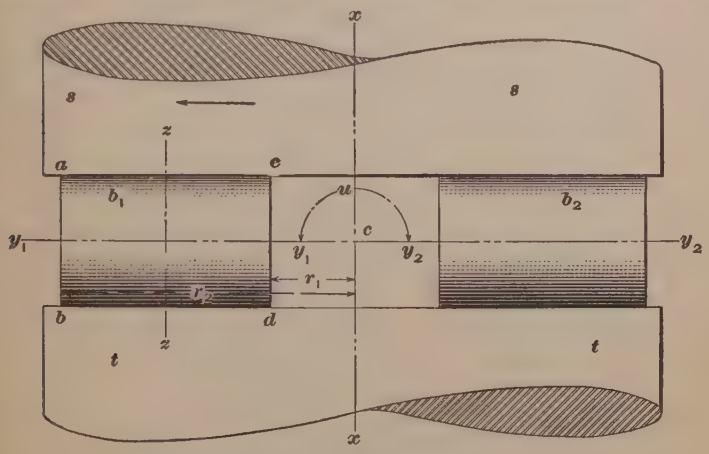


Fig. 27

at the point c, and the position of this point will be unaltered throughout a revolution of the shaft.

Owing to the difference in peripheral speed between the points a and e at the end of shaft s, it is evident that the end $a\,b$ of the roller b_1 tends to rotate at a greater speed than the end $d\,e$. As this is impossible, it follows that there must be relative motion between one end or both ends and the contact surfaces, which will result in friction. As the rollers are caged, they are compelled to revolve around the axis $x\,x$ and, simultaneously, to revolve around a vertical axis located anywhere between a and e, as at $z\,z$. This latter motion is termed *spinning*, and is one that must be eliminated as far as possible if the bearing is to be efficient and durable.

Fig. 28

Attempts have been made to obviate the injurious effect of spinning in thrust bearings, as in Fig. 27, by using a great

number of very short rollers having rounded edges, as shown in Fig. 28. The difference between the circumferential speeds at the two ends of the roller is then decreased to such an extent as to be of little consequence.

Fig. 29 shows such rollers inserted in a *cage*, or guiding plate *e*, of a thrust bearing. This cage is provided with rectangular openings and several series of rollers, as a_1, a_2, etc., and b_1, b_2, etc., every alternate series being so arranged relatively to the others that the paths of any two series do not coincide; consequently, the wear is more evenly distributed. The axes of all the series converge toward the common center *c* located on the axis of the shaft. The cage *e* rests on a raised edge, or flange, *d*.

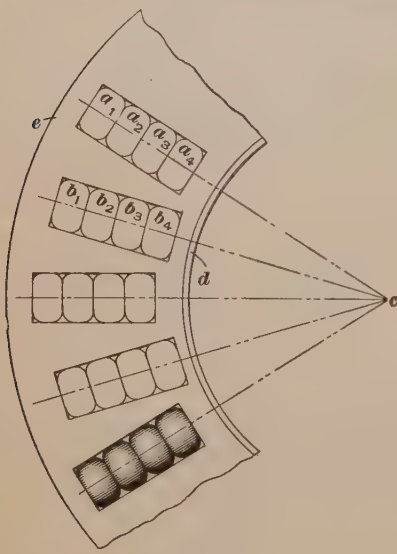


FIG. 29

Under certain conditions, thrust bearings of this kind have been very successful and have sustained very high pressures; both the rollers and the contact plates were made of hardened tool steel. Great care should be taken to see that the diameters of the rollers are perfectly uniform, and that the surfaces are absolutely plane and highly polished.

37. Conical Thrust Rollers.—If rollers having their axes at right angles to the axis of the shaft are to comply with the rules for true rolling, they must be made conical, and be proportioned according to the principles illustrated in Fig. 30 (*a*). That is, any roller constituting the frustum of a cone will have a true rolling motion if its apex coincides with the apexes of the conical surfaces on which it rolls.

The apex angles u_1, u_2 of these surfaces do not have to be equal; they may be varied until one of them becomes 180°, as at u_2, Fig. 30 (b), and may be increased beyond 180° if necessary.

It can be proved that if the roller b_1 is to have a true roll-

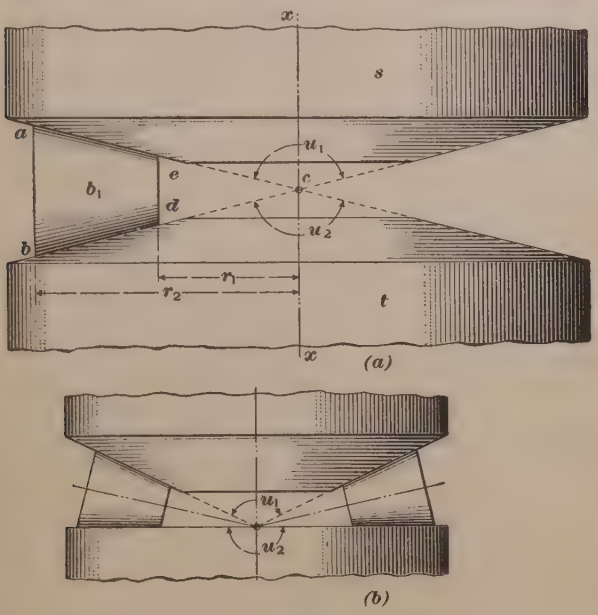


FIG. 30

ing motion the following proportion must be complied with:

$$a\,b : d\,e = r_2 : r_1$$

38. Axial Pressure of Conical Rollers.—In Fig. 31 (a), the shaft s is revolving in the direction of the arrow, and is supporting a load P that is transmitted to the bearing plate t by a series of rollers, two of which, b, b_1, are shown. Two forces P_1 and P_2 will therefore be acting normally to the sides of each roller. A resultant of these forces will act along the axis $c\,e$, tending to force the roller outwards; that is, away from the center c. Thus, a force P_3 will have to be applied at e, so as to prevent a longitudinal motion of the roller.

The magnitude of these forces may be ascertained as indicated in Fig. 31 (*b*). Draw the line *fg* parallel to *P*. Let P_0 represent the portion of the total load carried by each roller, and to some given scale, make *fg* of a length representing the magnitude of P_0. Draw *fh* at right angles to *fg*, and from point *g* line *gh* parallel to P_1. Then, the point of intersection *h* will determine the length of $gh = P_1$. As the action of P_1 is opposed by the reactive force P_2, draw

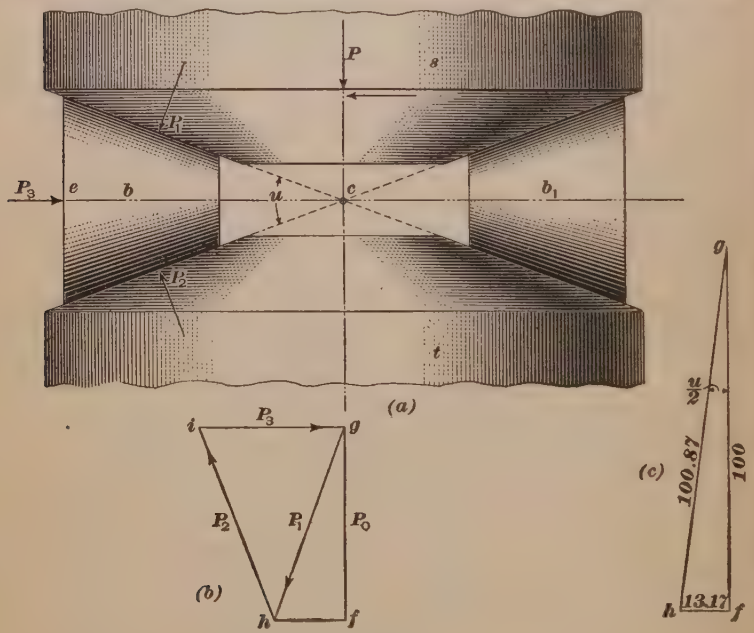


FIG. 31

the line *hi* parallel to the latter, and from *g*, the line *gi* parallel to the axis *cc*. Then, *gi* will indicate the magnitude of the reaction P_3 required to hold the roller in place. It is evident that this force P_3 should be as small as possible, and that, therefore, the angle *u* should be as small as the conditions will allow. This angle should not exceed 15°.

EXAMPLE.—In a roller thrust bearing similar to Fig. 31 (*a*), the pressure P_0 on each roller is 100 pounds. If the vertex angle *u* of the roller is 15°, what will be the pressures P_1 and P_2?

SOLUTION.—In solving the problem graphically, it is sufficient to draw the portion corresponding to fgh, Fig. 31 (b). Draw a line fg, Fig. 31 (c), representing the pressure of 100 pounds to any convenient scale, as, for instance, 1 in. = 10 lb. Therefore, $fg = 10$ in. Lay off the angle $fgh = \frac{1}{2}u = 7° 30'$, and draw gh; then, draw fh perpendicular to fg. By measuring the lines according to the chosen scale, it is found that the line $fh = \frac{1}{2}P_3 = 13.17$ lb., and, therefore, $P_3 = 26.34$ lb. The normal pressure $P_1 = gh = 100.86$ lb. Ans.

An example of a roller thrust bearing is shown in Fig. 32, which illustrates a full-sized transverse section through one side of the bearing. The bearing plate g revolves with the

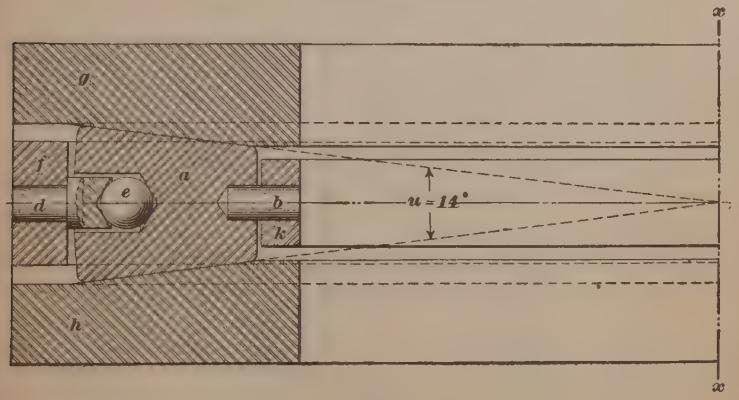


FIG. 32

shaft, and the plate h is stationary. One of the conical rollers is shown at a, and is held in position by the pins b and d. A ball e is inserted in a recess in the roller and rests against the concave end of the pin d, its purpose being to reduce the frictional resistance between the roller and the pin d. The inside pins are supported by the ring k, and the outside ones by the ring f, both rings revolving around the axis $x\,x$. In this instance, the apex angle u is 14°.

39. Most of the roller bearings manufactured come under one of the classes just described. On account of the rigidity of the parts, all bearings must be accurately shaped, and the material used must be extremely hard. In one form of roller bearing, the rollers are not solid, but are made

of steel bands wound spirally into the shape of a roller. Rollers constructed in this manner have a certain amount of flexibility, and thus require less accuracy in their manufacture; likewise, the surfaces against which the rollers revolve need not be of such extreme hardness.

40. Construction of Roller Bearings.—The strength of a roller bearing cannot be safely determined except by actual test; it depends to a great extent on the material used for the rollers and bearing surfaces. In some cases, bearings that are identical in size and construction will show great differences in durability owing to insufficient hardening of some of the parts. Calculations based on the strength of the material and the form of the rollers are not safe guides, as the actual strength is often found to be far below the estimated value. Extensive tests have been made by Striebeck, the results of which are embodied in the following formulas:

Let P = total load on bearing, in pounds;
P_1 = safe load for one roller;
l = length of roller, in inches;
d = diameter of roller, if cylindrical, and average diameter, if conical;
k = a coefficient which is 1,000 for hardened steel;
n = number of rollers in bearing.

Then, for a thrust bearing in which the load is supposed to be evenly distributed among the rollers,

$$P_1 = \frac{P}{n} \qquad (1)$$

For a cylindrical bearing in which the load acts in a direction normal to the journal axis, some of the rollers will carry the load while the rest are idle. Under these conditions, it is generally found that

$$P_1 = \frac{5P}{n} \qquad (2)$$

The safe load for one roller in either of these bearings is

$$P_1 = k\,d\,l \qquad (3)$$

41. Before constructing a roller bearing, it is necessary to ascertain the number of rollers required to support a given load. Having found this number, the diameters of the surfaces along which the rollers revolve must be calculated. Referring to Fig. 33, let

D_1 = diameter of circle on which the roller centers are located;

D_2 = diameter of circle tangential to the inner side of the rollers;

d = diameter of rollers;

C = diameter of circle circumscribing the rollers.

Then, $C = D_1 + d$ **(1)**

and $D_2 = D_1 - d$ **(2)**

From the center o, draw radii $o\,l$ and $o\,m$ to the centers of two adjoining rollers. Connect these centers with the chord $l\,m$, and draw a line $o\,n$ to the point of tangency between the two rollers. Then, the angle $l\,o\,m = \frac{360}{n} = 2u$, and the angle $m\,o\,n$ is

$$u = \frac{180}{n} \qquad \textbf{(3)}$$

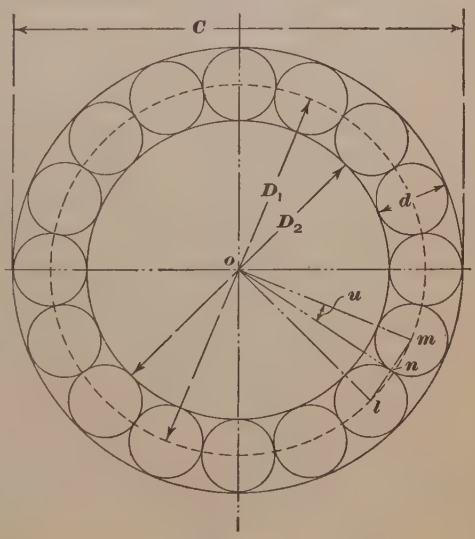


FIG. 33

In the right triangle $m\,n\,o$, the side $m\,n = m\,o \times \sin u$; and as $m\,o = \frac{1}{2} D_1$ and $m\,n = \frac{1}{2} d$, it follows that $\frac{1}{2} d = \frac{1}{2} D_1 \sin u$, or $d = D_1 \sin u$. Hence,

$$D_1 = \frac{d}{\sin u} \qquad \textbf{(4)}$$

Having found the value of D_1 by means of formula **4**, the values of C and D_2 may be found from formulas **1** and **2**.

These formulas are based on the assumption that all the

rollers are in contact with one another. In practice, the cage in which the rollers are inserted leaves a certain amount of space between each pair of rollers, and this space must be considered in using formula **4.**

Let i = space between two adjoining rollers;

d_1 = roller diameter plus one-half the space on both sides (which is equal to the space between two adjoining rollers), or $d + i$.

Then, $\frac{1}{2} d_1 = \frac{1}{2} D_1 \sin u$; $d_1 = D_1 \sin u$

and
$$D_1 = \frac{d_1}{\sin u} \qquad (5)$$

The value of the angle u, as found from formula **3,** is not affected by the addition of the space i. The values of C and D_2 for the new value of D_1 are still found by means of formulas **1** and **2.**

EXAMPLE.—A roller bearing is to contain twelve cylindrical rollers, each 1 inch in diameter. Assuming that there is to be a space of $\frac{1}{4}$ inch between each pair of rollers, find the diameter of the circle passing through the roller centers of the journals, and of the outside casing.

SOLUTION.—From formula **3,**

$u = \frac{180}{12} = 15°$, and $d_1 = d + i = 1 + .25 = 1.25$ in.

From formula **5,**

$$D_1 = \frac{1.25}{\sin 15°} = \frac{1.25}{.2588} = 4.83 \text{ in.}$$

From formula **1,**

$$C = D_1 + d = 4.83 + 1 = 5.83 \text{ in.}$$

From formula **2,**

$$D_2 = D_1 - d = 4.83 - 1 = 3.83 \text{ in.} \quad \text{Ans.}$$

BALL BEARINGS

42. Distinctive Features.—Fig. 34 (a) illustrates a cylindrical roller $a\,b\,c\,d$, having hemispherical ends. If the length of this roller is reduced until the lines $a\,b$ and $c\,d$ coincide, as in (b), a ball is produced. Hence, **ball bearings** may be considered as a special form of roller bearings in which the length of the linear contact is reduced to a point. In reality, owing to the elasticity of the material, the contact of a roller with its support does not constitute a line, but

rather a rectangular surface. Similarly, the contact between a ball and its support is not a point, but a circular or an elliptical surface, depending on the shape of the support. In all theoretical considerations, however, it is assumed that there is either line or point contact.

In reducing the length of a roller to that of a ball, certain advantages are gained. By reason of the line contact of the rollers in a roller bearing, they suffer from a certain lack of adjustability and from stringent requirements as regards the exactness of the engaging surfaces and the parallelism of their axes. If the journal should show signs of

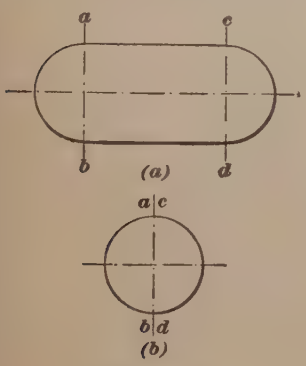


FIG. 34

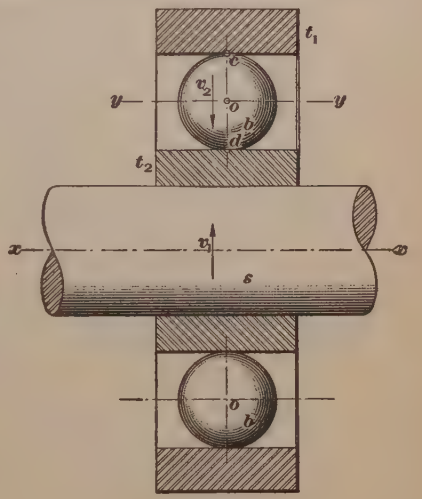


FIG. 35

deflection under load, it would materially affect the efficiency and durability of the bearing. In a ball bearing, the length of the line contact is reduced to a minimum; hence, the adjustability of the rolling parts is a maximum. If, in roller bearings, it is important that true rolling take place, and that material and finish be of the best, it is still more so in ball bearings, where the supporting area is more limited and therefore more sensitive to irregularities in relative motion and character of the contact surfaces.

43. Radial Bearings.—The fundamental form of a ball bearing is one in which the axes of rotation of the balls

are parallel with that of the journal, as shown in Fig. 35, in which s is the journal, t_1 the *outer race*, t_2 the *inner race*, and b, b the balls. The axis $x\,x$ of the journal is parallel with the axis of rotation $y\,y$ of any one ball. This bearing corresponds in principle to the roller bearing shown in Fig. 25, and is termed a **radial ball bearing,** because the two points of contact c and d lie on a line radial to the journal axis.

44. As a bearing of the form shown in Fig. 35 would not, without the use of supplementary means, prevent the balls from leaving their races, the latter are provided with grooves c_1 and c_2, as shown in Fig. 36. In cross-section, these grooves appear as circular arcs of radii r_1 and r_2, which are somewhat larger than the radius of a ball. Authorities differ as to the most suitable ratio between these radii, but Striebeck, from extensive experiments, recommends $r_1 = r_2 = \frac{2}{3}\,d$, where d is the diameter of a ball. These proportions are given the grooves shown in Fig. 36.

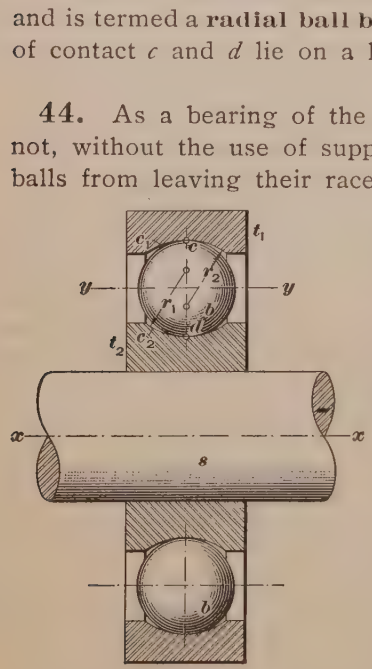


FIG. 36

In a bearing of this form, the journal is still allowed a certain amount of axial play by reason of the flatness of the curves in the grooves. This form possesses practically the same quality of pure rolling inherent in the form shown in Fig. 35, but has in addition the advantage of a greater surface contact, because the curvature of the supporting surface conforms more nearly to that of ball. Ball bearings of this class have proved very efficient, even under heavy pressures. If the curvature of the ball race is made as shown in Fig. 36, the end thrust of the journal may be as high as one-third of the load, normal to the journal axis, without requiring any provisions for preventing end motion.

45. Another variation of Fig. 35 is shown in Fig. 37 (*a*), in which one of the rolling surfaces, instead of being cylindrical, consists of two conical surfaces joined along the line *f g*. This variation is termed a **three-point bearing,** there being three contact points *c*, *d*, and *e*, which are points of tangency between the ball and the ball races. In this example, the balls partake of another motion besides pure rolling. This motion may be seen more clearly in Fig. 37 (*b*) and (*c*), where the angle between the sides of the groove is made more acute. If while rolling along the race t_1 the ball *b* turns in the direction of the arrow v_1, then, at any moment, the ball revolves around a line *z z*, passing through the points *c* and *d*, as an instantaneous axis of rotation. Theoretically, these places of contact should be merely points, but, actually, they are two circular surfaces with *c* and *d*, respectively, as centers. By looking along the line *i o*, the ball will appear as in Fig. 37 (*c*), revolving around the center *c* in the direction of the arrow v_2. The contact area at *c* may appear somewhat like the shaded circular surface, having a sliding friction similar to that found at the end of a revolving shaft resting on a plane surface.

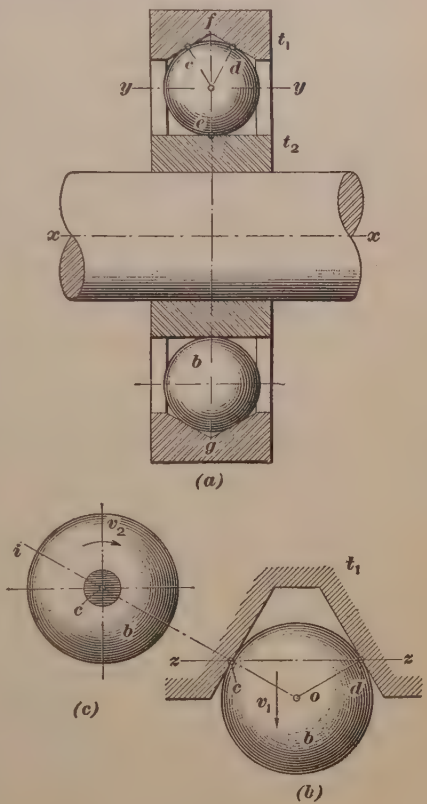


FIG. 37

This supplementary motion of the ball around a point of contact, termed spinning, should be absent from any bearing where high efficiency is required. The spinning motion of a ball is somewhat similar to that of a top, which revolves around its axis while it advances over the supporting surface.

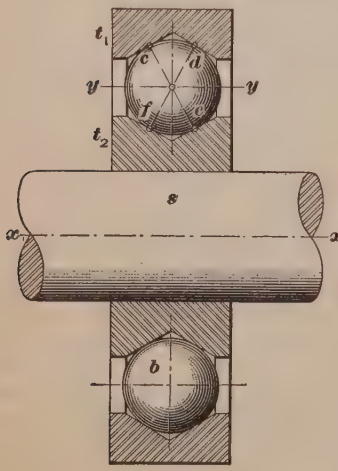


FIG. 38

46. If the race t_2 also is provided with a groove, as in Fig. 38, then a **four-point bearing** is produced, having contact point c, d, e, and f, at each of which spinning takes place.

47. Thrust Bearings.—In **thrust bearings**, the axis of rotation $y\,y$ of the ball is normal to the axis $x\,x$ of the journal, as shown in Fig. 39. This bearing corresponds to the roller bearing illustrated in Fig. 27. In this form, the balls will not retain their positions unaided, and a cage, or perforated plate, must therefore be used for the purpose. The balls while following a circular path with radius R will revolve, or spin, once around the axis $z\,z$ for each revolution around the axis $x\,x$.

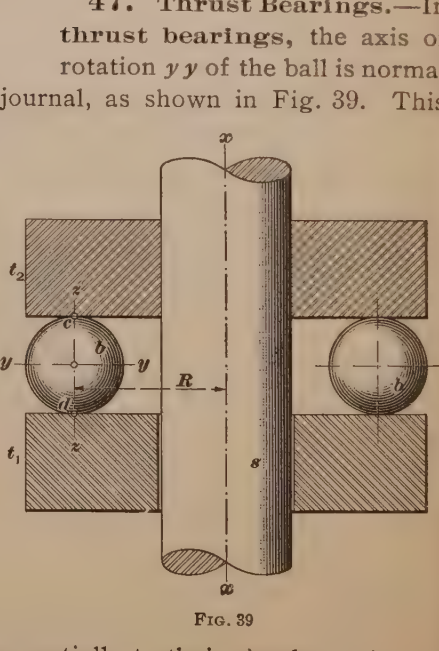


FIG. 39

48. To prevent the balls from running off tangentially to their circular path, one of the races, as shown in Fig. 40, is provided with a groove

having sloping sides. As here shown, the ball will not have a true rolling motion for the following reasons: It may be considered part of two separate cones, one of which has its vertex at z_1 and rolls along the plane cz_1, while the other has its vertex at z_2 and rolls along the plane dez_2. Evidently it is impossible for two cones revolving around different axes to be rigidly connected through the ball b; hence, there must be a resultant of the two motions, represented by a common axis of rotation, having one point at o and another point somewhere between z_1 and z_2, as at z. There will be spinning at all three points of contact. If the race t_1 causes an excess of resistance to relative motion between it and the ball, then the spinning at c will be increased and the point z will be nearer z_2.

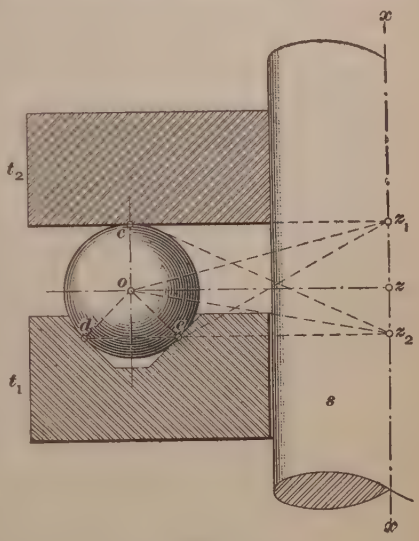


Fig. 40

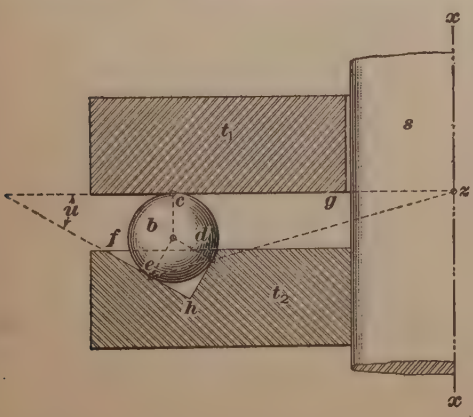


Fig. 41

49. Fig. 41 illustrates a rearrangement of Fig. 40, in compliance with the requirements for true rolling. The method of construction is as follows: If cg is to be one of the bearing surfaces, produce the line cg until it intersects

the axis $x\,x$ at z, which point will be the vertex of an imaginary cone. From z draw a line intersecting the ball b at two points, as d and e, which are to act as contact points between the ball and the race t_2. The sides $f\,h$ and $d\,h$ of the groove should be tangent to the ball at these points, as shown. While a certain freedom prevails in the choice of the contact points by varying the angle $c\,z\,e$, there is a position below which the point e must not fall. Experiments have shown that if the angle between any two adjoining surfaces, such as the angle u between the planes $e\,f$ and $c\,g$, are below a certain value, the ball tends to wedge itself between the surfaces. The angle should be at least 25°.

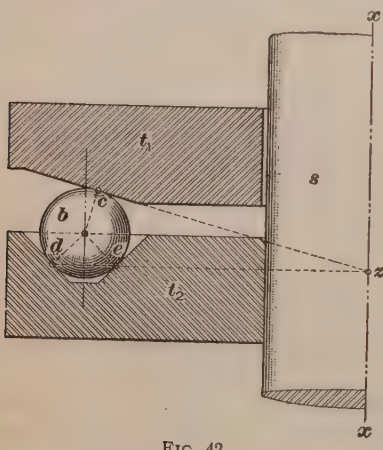


FIG. 42

On the other hand, the angle u should not be too large, as in that case the angle $f\,h\,d$ would become too acute. This would give the ball a tendency to wedge itself in the groove at h, thereby greatly increasing the pressure at the points d and e where spinning takes place.

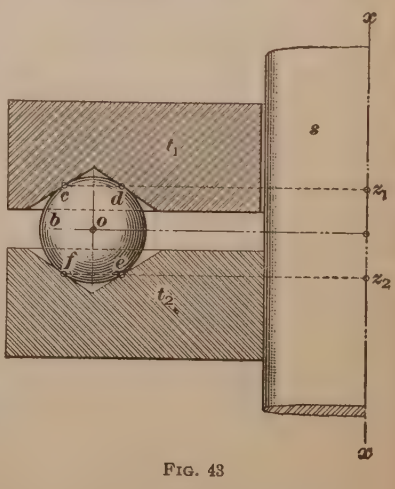


FIG. 43

50. The position of the vertex z may be changed without affecting the action of the ball. In Fig. 42, for instance, the point z has been lowered so as to make the lower side of the imaginary cone normal to the axes $x\,x$. If

the construction is made as explained in Art. 49, the conditions for practically true rolling are also here fulfilled.

51. Fig. 43 shows a form of ball bearing that has been extensively used in various machines, although it is very inefficient. In this instance, the lines cz_1 and fz_2, which intersect the ball at the points c, d, e, and f, are parallel. From the foregoing, it is clear that if the ball is supposed to be part of a cone, its sides should roll simultaneously along the surfaces cz_1 and fz_2; but as it cannot do so, there must be some sliding at one or more points, which should be avoided.

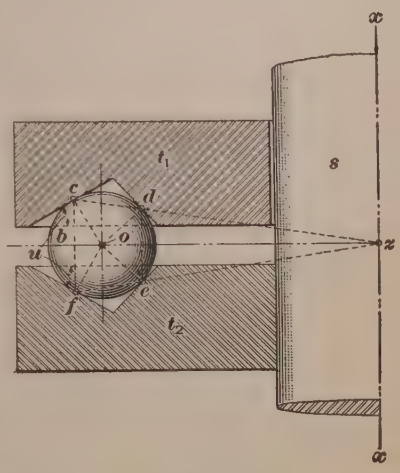


Fig. 44

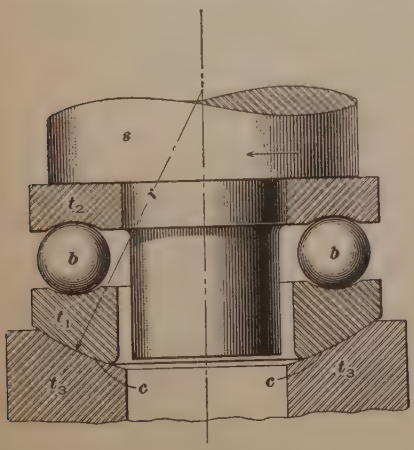


Fig. 45

52. In order to insure true rolling, the bearing should be constructed as shown in Fig. 44. When produced, the lines cd and ef passing through the four contact points should have a common point of intersection z located on the axis xx. Of course, the axis oz need not be normal to xx; the point z can be moved to some extent above or below its present position, but care must be taken that neither of the angles u, u is too small. In the diagram, each angle is 30°.

53. Fig. 45 shows a thrust bearing with ball races of the form shown in Fig. 36. The surface of plate t_1 is curved at c to the radius r, thus providing a certain amount of adjustability relative to the main bearing t_2. This is necessary if the load is to be evenly distributed among all the balls b. The plate t_2 is fastened to the shaft s.

54. Hub Bearings.—There is still another class of ball bearings, known as **hub bearings,** which may be considered as a combination of a radial and a thrust bearing. This type differs from the regular thrust bearing in that the end thrust does not proceed from without the bearing, but

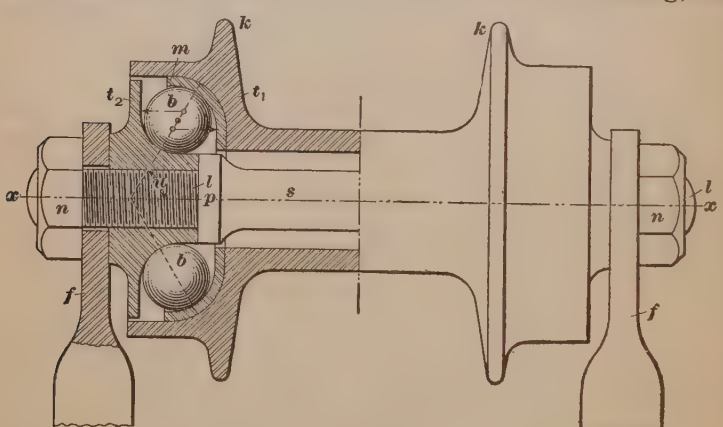


FIG. 46

from within, as a result of the arrangement of the ball races. These bearings are always used in pairs, so that the end thrust produced by one is neutralized by that produced by the other. The evolution of the bicycle served to bring this class of bearing into prominence.

In order that its action may be better understood, the type of hub bearing used in a bicycle wheel is shown in Fig. 46. The stationary spindle, or axle, s of a rear wheel has threaded ends l, l, which engage with nuts n, n and cones t_2 on each end. The cone shown is permanently screwed against the shoulder p; at the other end of s there is no shoulder, thus permitting the other cone to be moved longitudinally on l

in order that the bearings may be adjusted. By screwing the nuts n tightly against the rear forks f, f of the bicycle frame, they are forced against the cones, preventing them from turning and at the same time fastening the spindle to the fork. The balls b, b transmit the load supported by the spindle to the cups or ball races m inserted in each end of the hub t_1. The hub is provided with flanges k, k, to which are fastened the inner ends of the spokes, not shown, which connect the hub and the wheel rim.

55. Fig. 47 shows, diagrammatically, one end of a **two-point hub bearing,** t_1 being the hub, t_2 the cone, s the spindle, or axle, b the ball, and c and d the two contact points

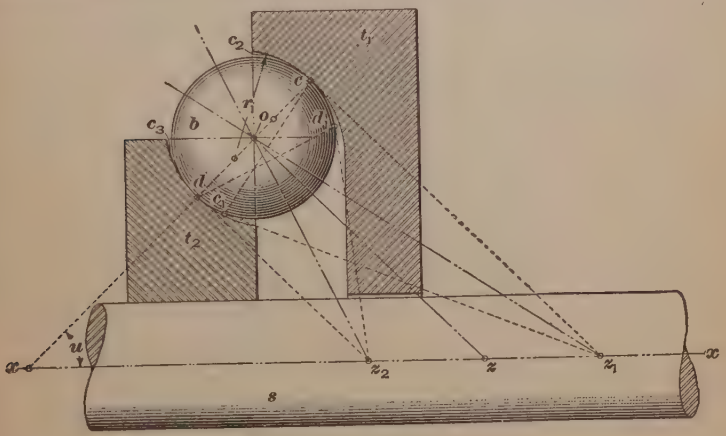


FIG. 47

with the races c_2 and c_3. The principles on which the action of this bearing is based are similar to those explained in Art. **48.** Drawing the tangents $c\,z_1$ and $d\,z_2$, and considering the ball as part of two cones with vertexes z_1 and z_2, respectively, it is again evident that true rolling cannot take place at both planes $c\,z_1$ and $d\,z_2$. In the present instance, the ball will revolve around an axis situated somewhere between $o\,z_1$ and $o\,z_2$, as, for example, at $o\,z$. Spinning will therefore occur at both c and d. In a bearing having the relative proportions shown in Fig. 47, it is found that for each revolution

of the hub t_1 around the axis $x\,x$, the ball will make about four-fifths revolution around the axis $c\,d$. The amount of this rotary motion will increase with an increase of the angle u.

The radius r, varies between nine-sixteenths to three-fourths of the ball diameter. On account of this curvature the bearing cannot be so closely adjusted that the rolling paths of the ball will be strictly defined. For this reason, the hub has a certain amount of longitudinal play. This play will increase with an increase of the angle u, which, in a bicycle, may be as much as 60°, because there the side pressure is of secondary importance.

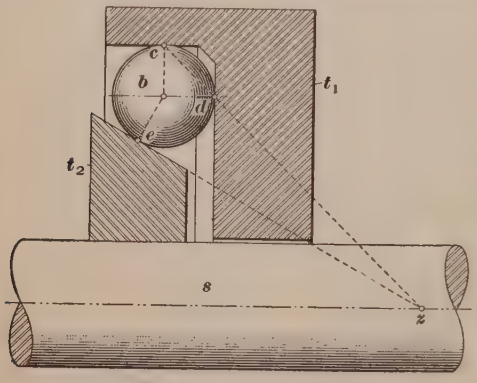


Fig. 48

56. Figs. 48 and 49 are diagrams illustrating **three-point** and **four-point bearings,** respectively. In Fig. 48, the contact points lie on the sides of the imaginary cone $c\,d\,z\,e$, and in Fig. 49, on the cone $c\,d\,z\,e\,f$. The method of construction is similar to that shown in Figs. 42 and 44.

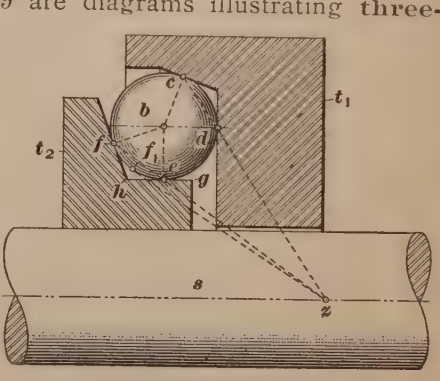


Fig. 49

57. By referring to the four-point bearings shown in Figs. 38, 44, and 49, it should be noted that these bearings require great accuracy in both their construction and adjustment in order to operate correctly. The reason for this may be seen, for instance, from Fig. 49. It is evident that

if the cylindrical surface $g\,h$ should have a radius somewhat smaller than shown, the ball would fail to make contact at e. The ball would still retain its position, but would make a three-point contact, the point f assuming the functions of the points e and f, but at a greatly increased pressure. It will be further observed that under these circumstances the position of point f is incorrect; it should occupy a position somewhere between points e and f, as at f_1, the point of tangency between a line $z\,f_1$ and the ball.

58. By comparing the two-, three-, and four-point bearings, it will be seen that in the last the balls are confined to a definite path, but that the bearing demands a very careful construction. In a two-point bearing, the path may vary on either bearing surface; hence, the bearing possesses a maximum of adjustability. A three-point bearing occupies an intermediate position, the rolling path being definitely determined for one surface, while that on the other has a certain amount of freedom.

59. Diameter of Ball Circle.—The diameter of the circle passing through all the centers o, Fig. 35, of the balls in a bearing is found, as described in Art. **41,** by means of formula **4.** This is under the supposition that the balls touch one another; but the conditions of easy running demand that a certain amount of clearance be left between the balls. The space left between each pair of balls varies with the total number of balls in the bearing. This space may be given a trial value of .005 inch, which is satisfactory if the sum of all the spaces does not exceed one-fourth of the ball diameter, otherwise the value should be reduced accordingly. The diameter of the ball circle is then found by means of formula **5,** Art. **41.**

EXAMPLE.—A ball bearing contains fourteen balls, each $\frac{1}{4}$ inch in diameter. Calculate the diameter of the ball circle if the balls are given suitable clearance.

SOLUTION.—If a clearance of .005 in. is given each pair of balls, the total clearance is $14 \times .005 = .07$ in.; as the value $\frac{d}{4} = \frac{.25}{4} = .0625$ in.,

the total clearance should be reduced to this amount. Therefore, the clearance for each pair of balls should be .0625 ÷ 14 = .0045 in.

From formula **5**, Art. **41**, $D_1 = \frac{d_1}{\sin u}$. Here, $d_1 = .25 + .0045 = .2545$ in. From formula **3**, Art. **41**, $u = \frac{180}{n} = \frac{180}{14} = 12° 51'$. Hence,

$$D_1 = \frac{.2545}{\sin 12° 51'} = \frac{.2545}{.22240} = 1.1443 \text{ in. Ans.}$$

60. Safe Load on Ball Bearings.—In thrust bearings of the type shown in Fig. 45, all the balls are supposed to take equal portions of the load, it being assumed that they are uniform in size. In bearings where the load acts in a direction normal to the axis of the journal, as in Figs. 36 and 46, the balls will be idle during one part of their travel around the journal and the load will be supported by two, three, or four balls, depending on the total number.

Let P = total load on bearing, in pounds;

P_1 = safe load for one ball;

n = number of balls in bearing, varying from ten to twenty;

d = diameter of the balls, in inches;

k = a coefficient that varies with the form of the bearing and the number of contact points. For hardened steel it has the following values: 400 for a straight-line race, as in Fig. 35; 700 for three- and four-point bearings; 1,400 for a two-point radial bearing, as in Fig. 36.

Then, $$P_1 = k d^2 \qquad (1)$$

For a thrust bearing,

$$P = n P_1 \qquad (2)$$

For a bearing in which the load acts normal to its axis,

$$P_1 = \frac{5 P}{n} \qquad (3)$$

For the radial type of bearings, as in Fig. 36, the load is independent of the speed if the latter is uniform and below 3,000 revolutions per minute. If sudden changes of load and speed take place, then the coefficient k should be lowered.

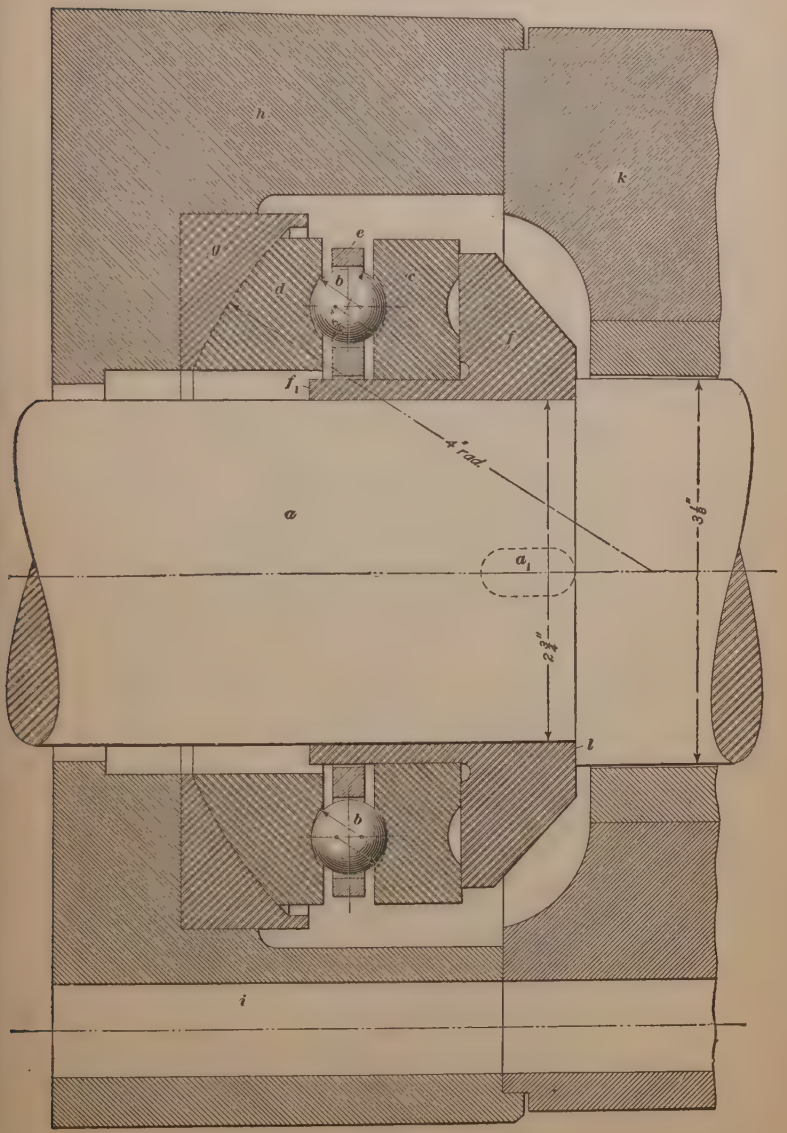


FIG. 50

EXAMPLE.—In a two-point ball bearing, as in Fig. 36, with fourteen balls each $\frac{1}{4}$ inch in diameter, the load is normal to the journal. Find the safe load.

SOLUTION.—From formula **1**,

$$P_1 = 1{,}400 \times .25^2 = 87.5 \text{ lb.}$$

From formula **3**, solving for P,

$$P = \frac{14 \times 87.5}{5} = 245 \text{ lb.} \quad \text{Ans.}$$

61. Examples of Ball Bearings.—Figs. 50 and 51 illustrate two types of ball bearings. They are not given as standards of design, but simply as examples of bearings that have worked efficiently under the conditions for which they were intended. Both are two-point bearings with grooves of the form shown in Fig. 36.

Fig. 50 shows a longitudinal section of one of the thrust bearings of a mine pump that was required to be in constant operation, the bearing sustaining an end thrust of about 2,000 pounds and the shaft revolving at a speed of 890 revolutions per minute. A pump with this style of bearing ran continuously for 289 days.

In the illustration, a is the pump shaft, which is driven by an electric motor, and b, b the $\frac{5}{8}$-inch balls, of which there are sixteen, placed in holes in the cage e. The two halves of the ball bearing are lettered c and d. In its earlier form the ring c rested directly on the shaft, but it was found that the intensity of pressure at the shoulder l was too great, resulting in a partial crushing of the material and a tendency of the ring to tip—hence, the insertion of the bushing f with the extension f_1, and the key a_1. The end surface of the other ring d is made spherical to allow it a certain amount of adjustment; it rests against the concave side of the ring g. The casing h of the bearing is bolted to the main body k of the pump by bolts inserted through holes i.

62. In Fig. 51 is shown a pedestal bearing with fifteen balls, each $\frac{7}{8}$ inch in diameter, supporting a shaft $2\frac{9}{16}$ inches in diameter; (a) is a longitudinal section and (b), a side view, partly in section, the section being taken along the line $x\,x$ in (a). In this bearing, tne balls b are not evenly

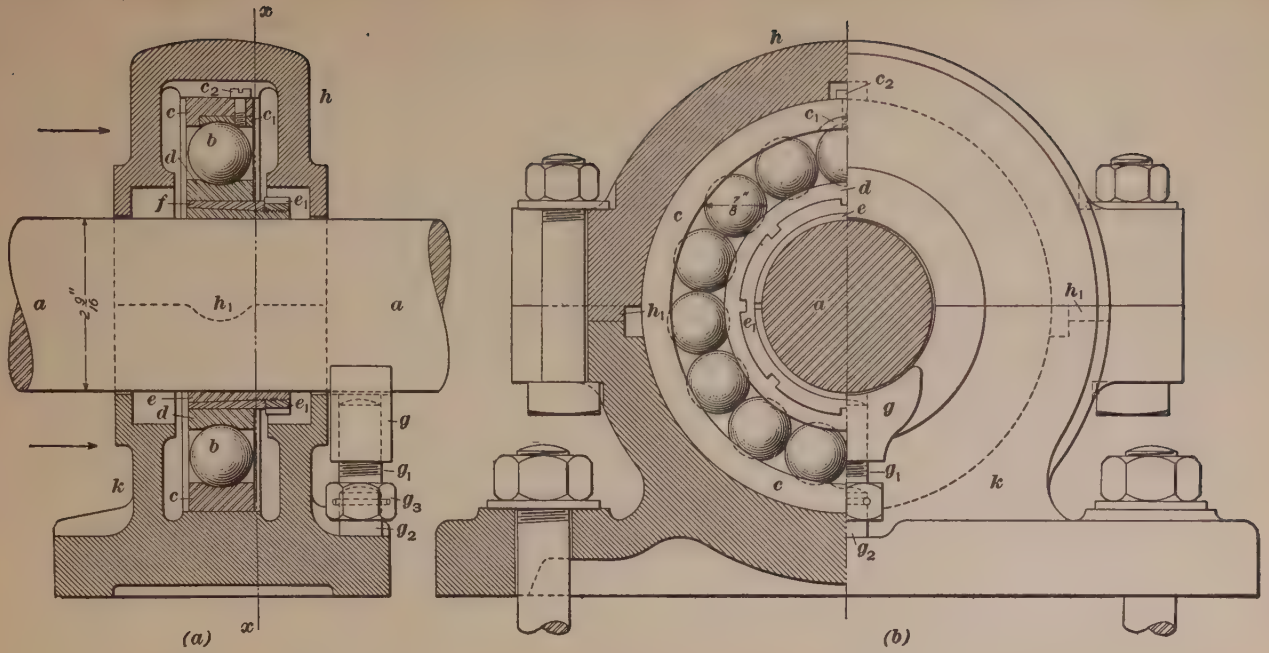

x
h
c2
c
c1
b
d
f
e1
a
2"
h1
a
e
d
e1
g
b
g1
k
c
g3
g2
x
(a)
h
c2
c1
d
e
c
g
a
h1
h1
e1
g
g1
c
k
g2
(b)

FIG. 51

loaded, being idle while passing over the upper side of the shaft. The ball races are shown at c and d, the latter resting directly on the shaft if there is little vibration present. Otherwise, this race is fastened in the manner shown in (a). The sleeve f has a conical hole into which is fitted the split conical bushing e, one end of which is threaded to engage with the ring e_1. By turning up ring e_1, the bushing is drawn outwards, locking the ring securely to the shaft.

As rings c and d are not split, some means must be provided for the insertion or removal of the balls. For this purpose, a recess large enough to allow a ball to pass is made in the ring c. Into this recess is fitted a piece c_1, which is held by the screw c_2. The recess should be located at a place where the piece c_1 will not be subjected to any pressure from the balls. The outer casing is divided into two parts h and k, and is held together by bolts as indicated, the tongues h_1 insuring the correct alinement of the two parts. Between the casing h and the ring c there is a play of .004 inch, to allow for longitudinal adjustment of the rings and for expansion in case the parts become warm.

The chair g serves the purpose of supporting the shaft while the bearing is being inspected. The threaded end of the bolt g_1 is screwed into the chair, while the head revolves loosely on the stud g_2, to which it is connected by a pin, as indicated. By turning the head g_3, the chair may be raised until it supports the shaft, thus permitting the removal of the ball races.

In a more recent form of this bearing, the piece c_1 is omitted and the ring c is made solid throughout. To permit the insertion of the balls, their number is decreased, and they are inserted while the ring c is placed eccentrically relative to d. After the ball races are arranged concentrically, elastic distance pieces are inserted between each pair of balls, thus spacing them evenly around the shaft.

It is important that both roller and ball bearings be incased in such a manner as to retain a lubricant and exclude grit and dust. The lubricant will also prevent rust, but it should be free from acid.

MACHINE DESIGN

Serial 997D (PART 4) Edition 1

SHAFTS, COUPLINGS, AND SPRINGS

SHAFTS

VARIETIES OF STRESSES IN SHAFTS

1. Shafts may be divided into three classes, according to the kinds of stresses to which they are subjected:

1. Shafts subjected chiefly to torsion or twisting, as, for example, line shafting in mills and shops, and, in general, shafts used to transmit power.

2. Shafts subjected chiefly to bending action, such as the axles of gears, etc.

3. Shafts subjected to both twisting and bending, such as engine crank-shafts.

When a shaft is intended simply to transmit power from one point to another, it does so by means of its resistance to torsion. Conditions under which a shaft is subjected to torsion alone exist only when it, if horizontal, is supported by a sufficient number of bearings, so as to prevent any bending tendency; also, that no pulleys or gears are supported by the shaft subjecting it to bending stresses. If the shaft is vertical, its weight must be supported by thrust bearings along its length, to prevent a tendency to buckle and thus to prevent bending stresses.

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In order that the turning force shall not be accompanied by bending forces at the place where power is transmitted to the shaft, it is necessary that the driving power be applied at least at two points diametrically opposite each other, as in Fig. 1. This illustration represents, diagrammatically, an overhanging shaft a projecting beyond the bearing g, and having at its end the pulley b as a driving member. P, P are the turning forces acting at the distance R from the center of the shaft. The turning moment T is therefore $P \times R + P \times R = P \times 2R$.

Turbines and electric motors are examples of machines by means of which a shaft may be driven by torsion alone; that is, without bending, as the turning forces are applied

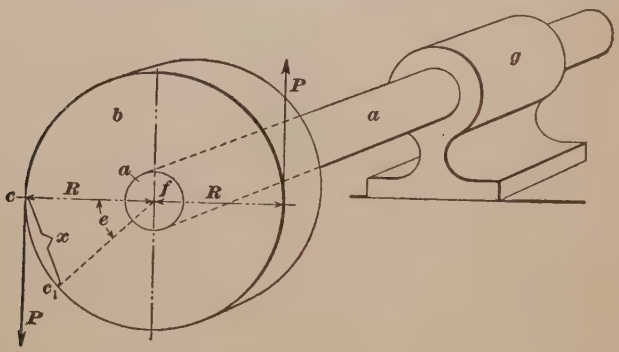

Fig. 1

at numerous points on the circumference of the revolving member.

2. When the turning force is applied at one point only —at one side of the shaft—as is the case with most machines, then the shaft is exposed not only to torsion, but also to bending. An example coming under this class is the combination of a connecting-rod and a crank-shaft. Another example, but one in which the turning and bending forces are constant for each revolution, is that of a belt and pulley, as shown in Fig. 2. Here, T_1 represents the tension of the tight and T_2 that of the slack side of the belt. The turning force P is equal to their difference, or $P = T_1 - T_2$.

In this case, the force P not only tends to twist the shaft, but the combined tensions T_1 and T_2 also tend to bend it. The twisting moment T is PR, when R is the radius of the pulley.

If the pulley shaft is overhanging, as in Fig. 1, the bending moment M varies from zero at the pulley to a maximum at the supporting bearing. If the pulley is located between two bearings, then the bending moment is a maximum at the pulley.

In Fig. 3, a gear is attached to the shaft and driven by

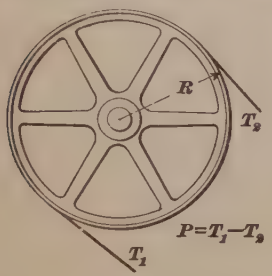

FIG. 2

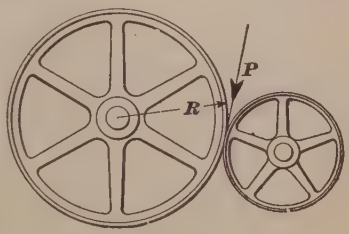

FIG. 3

another gear, in which case P will act at the pitch circle with a leverage equal to the radius R.

SHAFTS SUBJECTED TO TORSION

3. Moments of Inertia.—In the case of pure torsion, the stresses produced are those of shearing, the plane of shearing being at right angles to the axis of rotation. The moment of inertia found with reference to this axis, termed the **polar moment of inertia,** is generally denoted by the letter J, and has been referred to in *Strength of Materials*, Part 2. If I is the *rectangular moment of inertia*, then for a solid circular section,

$$J = 2I = 2 \times \frac{\pi d^4}{64} = \frac{\pi d^4}{32}$$

4. Torsional Strength of a Solid Round Shaft.—The ultimate resistance of a beam to bending must be equal to

the bending moment; or, according to the formula given in *Strength of Materials*, Part 2,

$$M = S_b \frac{I}{c}$$

If $I = \frac{\pi d^4}{64}$, and S_s is the safe fiber stress in flexure, the resisting moment of a round shaft will be

$$S_s \frac{\frac{\pi d^4}{64}}{\frac{d}{2}} = S_s \frac{\pi d^3}{32}$$

or,
$$M = \frac{S_s d^3}{10.2} \qquad (1)$$

The resisting moment of a shaft to twisting is found by a formula similar to formula **1**, except that the polar moment of inertia J is substituted for the rectangular moment of inertia I.

Let T = twisting moment, in inch-pounds;
P = turning force, in pounds;
R = length of lever arm of turning force, in inches;
S_s = ultimate shearing strength of shaft, in pounds;
S_s = safe shearing strength of shaft, in pounds;
J = polar moment of inertia of cross-section of shaft about neutral axis;
c = distance from neutral axis to outermost fiber;
f = factor of safety.

Then, the moment that resists twisting is $\frac{S_s J}{c}$, and the safe moment is $\frac{S_s J}{fc} = \frac{S_s J}{c}$, since $S_s = \frac{S_s}{f}$. Therefore,

$$T = \frac{S_s J}{c}$$

Inserting the value given for J in Art. **3**, and that of $c = \frac{d}{2}$, then the resisting moment is

$$S_s \frac{\frac{\pi d^4}{32}}{\frac{d}{2}} = S_s \frac{\pi d^3}{16}$$

Equating the twisting and resisting moments,

$$T = PR = S_s \frac{\pi d^3}{16} = \frac{S_s d^3}{5.1} \qquad (2)$$

whence $$d = \sqrt[3]{\frac{5.1\,PR}{S_s}} = 1.72\sqrt[3]{\frac{PR}{S_s}} \qquad (3)$$

Instead of using the twisting moment PR, it may be more convenient at times to use the horsepower transmitted by the shaft and the number of revolutions per minute. From the formula referring to keys in *Machine Design*, Part 2, where H is the horsepower and N the number of revolutions per minute,

$$T = PR = 63{,}025\,\frac{H}{N}$$

Substituting this value of PR in formula **3**,

$$d = 1.72\sqrt[3]{\frac{PR}{S_s}} = 1.72\sqrt[3]{\frac{63{,}025\,H}{NS_s}}$$

or, $$d = 68.45\sqrt[3]{\frac{H}{NS_s}} \qquad (4)$$

The values for S_s may be taken as 3,400 for cast iron, 6,800 for wrought iron, and 9,000 for steel. Ordinarily, these values are the ones that are used in good practice. Under exceptional circumstances, it may be necessary to use lower values, particularly if the shaft is subjected to shocks or if the stress changes suddenly or violently.

5. Angle of Torsion, or Twist, of a Solid Round Shaft.—As shown in *Strength of Materials*, Part 2, the angle of twist is directly proportional to the twisting moment, if the material is not strained beyond the elastic limit. Let the angle of torsion be indicated in Fig. 1 by the angle $cfc_1 = e$. If the amount of twist, or distortion, $cc_1 = x$ is produced in a length l of the shaft, it can be proved by higher mathematics that

$$\frac{x}{l} = \frac{S_s}{E_s} \qquad (1)$$

In this formula, E_s is the modulus of torsional elasticity, or of elasticity in shear. E_s may also be said to be the ratio

between the unit torsional shearing stress and the strain, or distortion, per unit length, or

$$E_s = \frac{S_s}{\frac{x}{l}} = \frac{S_s l}{x}$$

The circumference of the shaft is πd, and the angle of twist is e. The arc included by any angle e is $x = \frac{\pi d e}{360}$. Inserting this value of x in formula **1**,

$$\frac{\pi d e}{360 l} = \frac{S_s}{E_s}, \text{ hence } e = \frac{360 S_s l}{\pi d E_s}$$

From formula **2**, Art. **4**, $T = \frac{S_s d^3}{5.1}$ and $S_s = \frac{5.1 T}{d^3}$. Substituting this value in the formula $e = \frac{360 S_s l}{\pi d E_s}$,

$$e = \frac{360 \times \frac{5.1 T l}{d^3}}{\pi d E_s}$$

or,

$$e = \frac{584 T l}{E_s d^4} \qquad (2)$$

The value of E_s for steel is about 11,000,000, for cast iron 6,000,000, and for wrought iron 10,000,000. If the horsepower transmitted by the shaft and the number of revolutions per minute are given, then by substituting the value of $T = 63{,}025 \frac{H}{N}$ in formula **2**,

$$e = \frac{36{,}806{,}600 H l}{N E_s d^4} \qquad (3)$$

If the shaft is made of steel, formula **2** will be

$$e = \frac{.0000531 T l}{d^4} \qquad (4)$$

and

$$T = \frac{e d^4}{.0000531 l} \qquad (5)$$

Substituting the value $E_s = 11{,}000{,}000$ in formula **3**,

$$e = \frac{3.346 H l}{N d^4} \qquad (6)$$

EXAMPLE.—A 2-inch steel shaft 20 feet long transmits 25 horsepower at a speed of 120 revolutions per minute. Find the angle of torsion of the shaft.

SOLUTION.—By formula 6,

$$e = \frac{3.346 \times 25 \times 240}{120 \times 16} = 10.46°. \quad \text{Ans.}$$

6. Limits of the Angle of Torsion.—The diameter found by means of formula **4**, Art. **4**, will give the shaft sufficient *strength*, but it may be deficient in torsional *stiffness*. Under these conditions, a long shaft of small diameter may act as a spring, with the result that one or both ends will revolve at a variable velocity. This is objectionable when the conditions are such that a shaft should run at a constant velocity under variable loads. It is therefore customary to add from $\frac{3}{8}$ to $\frac{5}{8}$ inch to the diameter found according to formula **4**, Art. **4**, in order to increase the stiffness of the shaft and to prevent vibration.

More accurate results may be obtained by calculating the angle of torsion, which should not exceed 1° for a length of shaft equal to twenty times the diameter. Testing the results found in the example of Art. **5** by this rule, the angle of torsion should not exceed $\frac{l}{20\,d} = \frac{240}{20 \times 2} = 6°$. As the angle is 10.46°, the diameter should be increased to give greater stiffness.

The foregoing statement may be incorporated directly into a formula by inserting the values $e = 1°$, and $l = 20\,d$ in formula **5**, Art. **5**. Then,

$$T = \frac{d^4}{.0000531 \times 20\,d} = \frac{d^3}{.00106}$$

and

$$d^3 = .00106\,T$$

Whence

$$d = .102\sqrt[3]{T} \qquad (1)$$

If the horsepower to be transmitted by the shaft is known, then the value $T = 63{,}025\,\frac{H}{N}$ may be inserted in formula **1**, and

$$d = 4.059\sqrt[3]{\frac{H}{N}} \qquad (2)$$

Recalculating the example given in Art. **5** by formula **2**,

$$d = 4.059\sqrt[3]{\frac{25}{120}} = 2.406$$

or,

$$d = 2\tfrac{13}{32}, \text{ say } 2\tfrac{7}{16}, \text{ inches}$$

By comparing formula **2**, Art. **4**, with formula **5**, Art. **5**, it will be seen that for *strength* the diameter of a shaft is proportional to the *cube root* of the twisting moment, and that for *stiffness* the diameter of a shaft is proportional to the *fourth root* of the twisting moment.

7. Torsional Strength of a Hollow Shaft.—For a hollow round shaft, $J = \frac{\pi(d_1^4 - d_2^4)}{32}$, d_1 and d_2 being the outside and inside diameters, respectively. Inserting this value of J in the formula $T = \frac{S_s J}{c}$ and making $c = \frac{d_1}{2}$, then

$$T = PR = \frac{S_s \frac{\pi(d_1^4 - d_2^4)}{32}}{\frac{d_1}{2}}$$

or,
$$T = \frac{S_s(d_1^4 - d_2^4)}{5.1\, d_1} \qquad (1)$$

If the ratio $\frac{d_2}{d_1}$ is represented by n, then $n = \frac{d_2}{d_1}$ and $d_2 = n\, d_1$. Inserting this value of d_2 in the preceding formula,

$$T = PR = \frac{S_s[d_1^4 - (n\, d_1)^4]}{5.1\, d_1}$$
$$= \frac{S_s\, d_1^4(1 - n^4)}{5.1\, d_1}$$
$$= \frac{S_s\, d_1^3(1 - n^4)}{5.1}$$

from which
$$d_1 = 1.72\sqrt[3]{\frac{PR}{S_s(1 - n^4)}} \qquad (2)$$

If the horsepower transmitted by the shaft is given, then, by following the method given in Art. **4**,

$$d_1 = 68.45\sqrt[3]{\frac{H}{S_s N(1 - n^4)}} \qquad (3)$$

EXAMPLE.—A hollow steel shaft is to transmit 2,000 horsepower at 160 revolutions per minute. If $S_s = 9{,}000$ pounds, and the ratio between the inside and outside diameters is as 1 : 2, what will be the outside diameter?

SOLUTION.—Substituting these values in formula **3**,

$$d_1 = 68.45\sqrt[3]{\frac{2{,}000}{9{,}000 \times 160\,(1 - .5^4)}} = 7.803, \text{ or } 7\tfrac{13}{16}, \text{ in., nearly. Ans.}$$

8. Angle of Torsion of a Hollow Round Shaft.—The angle of torsion of a hollow round shaft may be found by means of a formula resembling formula **2**, Art. **5**, the only difference being that the value $d_1^4 - d_2^4$ is substituted for d^4. Thus,

$$e = \frac{584\,Tl}{E_s(d_1^4 - d_2^4)} \qquad (1)$$

Similarly, formula **3**, Art. **5**, changed so as to apply to hollow shafts, will be

$$e = \frac{36{,}806{,}600\,Hl}{NE_s(d_1^4 - d_2^4)} \qquad (2)$$

9. Ratio of Strength of Solid and Hollow Shafts. A close study of formulas **2** and **3**, Art. **7**, shows that the strength of a shaft is not seriously diminished by removing the central part. For instance, if the ratio of $d_2 : d_1 = n = \frac{1}{2}$, then the factor $1 - n^4$ becomes $1 - \frac{1}{16}$, showing that the strength of the shaft has been reduced by only $\frac{1}{16}$, or that the ratio of the strength of the solid to that of the hollow shaft is 1 : .9375. Similarly, if $n = .6$, then $1 - n^4 = .8704$, indicating that the ratio is as 1 : .87.

10. In a solid shaft, the poorest material and that which is least effective in resisting torsion is usually found at the center. In a hollow shaft, however, this material is removed, thereby increasing the average strength of the material that remains. Also, by forging shafts of large diameter hollow, the material is more thoroughly worked and will consequently have greater strength. Hollow steel shafts are much used for marine engines.

11. A hollow shaft will have a strength equal to that of a solid shaft if the moduli of the two sections are equal, or if formula **2**, Art. **4**, is equal to formula **1**, Art. **7**. That is.

$$\frac{S_s d^3}{5.1} = \frac{S_s(d_1^4 - d_2^4)}{5.1\,d_1}$$

or,

$$d^3 = \frac{d_1^4 - d_2^4}{d_1} \text{ and } d_1 = \frac{d_1^4 - d_2^4}{d^3}$$

If $d_2 = n\,d_1$, then $d_1 = \dfrac{d_1^4 - (n\,d_1)^4}{d^3}$ and $d_1^3 = \dfrac{d^3}{1 - n^4}$.

Or, $$d_1 = d\sqrt[3]{\frac{1}{1 - n^4}}$$

EXAMPLE.—A solid shaft has a diameter of 9 inches and is to be replaced by a hollow shaft of equal strength in which the ratio $d_2 : d_1 = n = .4$. Find the outside diameter.

SOLUTION.—By substituting these values in the formula,

$$d_1 = 9\sqrt[3]{\frac{1}{.9744}} = 9.078, \text{ or } 9\tfrac{1}{8}, \text{ in. Ans.}$$

12. Line Shafting.—Line shafting is a term applied to the long and continuous lines of shafting used in mills, factories, and shops for the distribution of power. The shaft is principally strained by torsion, but, in addition, there is always a bending action due to the weight of the shaft itself and the pulleys carried by it, and to the tension of belting or the thrust of gearing.

In calculating the diameter of a shaft for a given twisting moment, two things must be considered: *strength* and *stiffness*. Very large shafts or short shafts need be calculated for strength only; but in long lines of shafting of small diameter attention must be paid to stiffness and rigidity.

13. Speed of Line Shafting.—The speed of a shaft is fixed largely by the speed of the driving belt or by the diameters of the pulleys on it. In general, machine-shop shafts run from about 120 to 150 revolutions per minute, shafts driving woodworking machinery from about 200 to 250 revolutions per minute, while in cotton mills, the practice is to make the shaft diameter smaller and to run at a higher speed. Line shafts should generally not be less than $1\frac{3}{4}$ inches in diameter.

14. Distance Between Bearings.—The distance between bearings should not be great enough to permit a deflection of more than $\frac{1}{100}$ inch per foot of length. Hence, when the shaft is heavily loaded with pulleys, the bearings must be closer than when it carries only a few pulleys. Pulleys that give out a large amount of power should be placed as near to hangers as possible.

In Table I are given the maximum distances between the bearings of different sizes of continuous shafts used for the transmission of power.

TABLE I

MAXIMUM DISTANCE BETWEEN BEARINGS OF CONTINUOUS SHAFTS

Diameter of Shaft Inches	Distance Between Bearings Feet		Diameter of Shaft Inches	Distance Between Bearings Feet	
	Wrought-Iron Shaft	Steel Shaft		Wrought-Iron Shaft	Steel Shaft
2	11	11.5	6	19	20
3	13	13.75	7	21	22.25
4	15	15.75	8	23	24
5	17	18.25	9	25	26

SHAFTS SUBJECTED TO BENDING

15. The axles of large waterwheels, gear-wheels, etc. are loaded transversely and are not generally subjected to torsional stresses. They may therefore be treated as beams transversely loaded, and the bending stresses in the same may be ascertained analytically or by a combination of analytical and graphical methods.

When calculating bending moments for shafts and axles, it is customary to consider the center of the bearing as the center of moments, on the supposition that the fit between the journal and its bearings is of such nature as not to restrain a possible deflection of the journal. The strength of the shaft may therefore be calculated as if the shaft simply rested on two supports at the centers of the bearings. While this is not entirely correct, the supposition is on the safe side. In cases where the shaft supports a pulley, gear, etc., the latter being driven tight on the shaft, it is assumed that the bending stresses do not go much beyond the edge of the hub. To allow for a sufficient margin of safety in this case, it is also supposed that the center of the bending

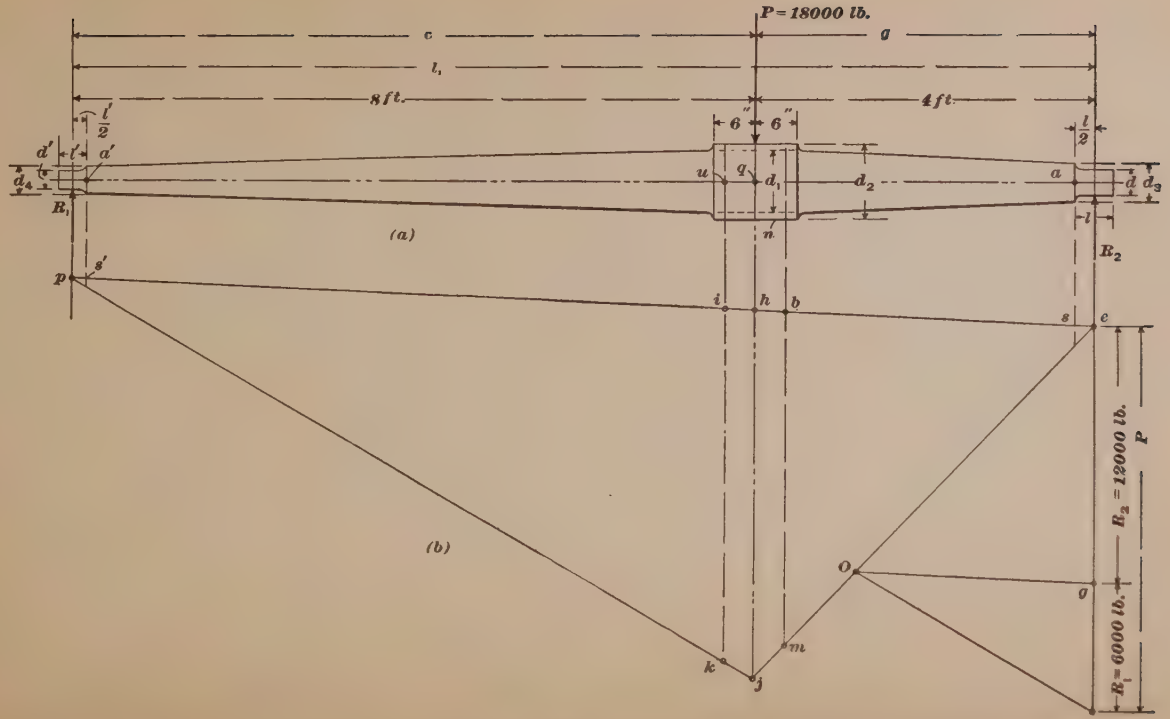
P = 18000 lb.
c
a
l
8 ft.
4 ft.
6″
6″
l′/2
l/2
d′
t′
a′
d₄
R₁
(a)
u
q
d₁
d₂
a
d
d₃
n
l
R₂
p
s′
i
h
b
s
c
(b)
O
g
k
j
m
R₂ = 12000 lb.
P
R₁ = 6000 lb.

FIG. 4

moment is located at the center of the hub, though some designers prefer to locate it nearer the end of the hub.

16. Axles Supporting One Load.—The methods used in finding the reactions and bending moments for axles supporting one load are best shown by an example.

EXAMPLE.—An axle 12 feet long between centers of bearings carries a wheel weighing 9 tons. The wheel is 4 feet from one end of the shaft and the length of its hub is 12 inches. The axle is of wrought iron. Required, the dimensions.

NOTE.—Two solutions of this problem will be given, the first wholly analytical and the second partly graphical and partly analytical.

SOLUTION 1.—Let Fig. 4 (*a*) represent, temporarily, the axle in question, P being the load of 9 tons at the distance indicated from either bearing, and R_1 and R_2 the reactions. To find the value of the reactions, proceed as follows:

Take, for instance, the center of the right-hand bearing as the center of moments and find the reaction at the center of the other bearing. Then,

$$P \times g = R_1 \times l_1 \text{ and } R_1 = \frac{P g}{l_1} = \frac{18{,}000 \times 48}{144} = 6{,}000 \text{ lb.}$$

$$R_2 = P - R_1 = 18{,}000 - 6{,}000 = 12{,}000 \text{ lb.}$$

The resisting moment of a beam is $S_t \frac{I}{c}$, and since the safe stress S_t remains constant for the same beam, the resisting moment is proportional to $\frac{I}{c}$. For a beam of circular section,

$$\frac{I}{c} = \frac{\frac{\pi d^4}{64}}{\frac{d}{2}} = \frac{\pi d^3}{32}$$

Hence, since $\frac{\pi}{32}$ is a constant quantity, the resisting moment at any section of a circular shaft is proportional to the cube of the diameter at that section. Conversely, the diameter of the shaft at any section must be proportional to the cube root of the resisting moment at that section, and to have the shaft of equal strength throughout, the diameter at each section should be proportional to the cube root of the *bending moment* at that section.

In finding the diameter d_1, the bending moment is taken at the point q, and is $R_1 c = 6{,}000 \times 96$. For ascertaining the diameter at this point, use formula **1**, Art. **4**,

$$M = R_1 c = \frac{S_t d_1^3}{10.2}$$

from which

$$d_1 = \sqrt[3]{\frac{10.2 \times K_1 c}{S_t}}$$

Assuming a safe fiber stress $S_s = 8{,}500$,

$$d_1 = \sqrt[3]{\frac{10.2 \times 6{,}000 \times 96}{8{,}500}} = 8.842, \text{ or } 8\tfrac{7}{8}, \text{ in., nearly}$$

The shaft should be enlarged at this place to allow a keyway to be cut without weakening the shaft. According to the formula given in *Machine Design*, Part 2, the depth of the sunk key is $\frac{1}{6}\,d_1 = 1\frac{1}{2}$ in. The depth of the keyway is, therefore, $\frac{3}{4}$ in., and the diameter d_2 of the enlarged portion should be at least $8\frac{7}{8} + 1\frac{1}{2} = 10\frac{3}{8}$ in.

The lengths and diameters of the journals may now be calculated. For the journal nearest the load, from the formula given in *Machine Design*, Part 3,

$$d = 1.5\sqrt{\frac{R_2}{\sqrt{S_s\, p}}}$$

Assuming that $p = 700$,

$$d = 1.5\sqrt{\frac{12{,}000}{\sqrt{8{,}500 \times 700}}} = 3\tfrac{3}{8} \text{ in., nearly}$$

$$l = \frac{R_2}{p\,d} = \frac{12{,}000}{700 \times 3\frac{3}{8}} = 5\tfrac{1}{8} \text{ in., nearly}$$

For the other journal,

$$d' = 1.5\sqrt{\frac{R_1}{\sqrt{S_s\, p}}} = 1.5\sqrt{\frac{6{,}000}{\sqrt{8{,}500 \times 700}}} = 2\tfrac{3}{8} \text{ in.}$$

$$l' = \frac{6{,}000}{700 \times 2\frac{3}{8}} = 3\tfrac{5}{8} \text{ in.}$$

$d_4 = d' + 2\,e$, where $e = \frac{1}{10}\,d' + \frac{1}{8} = .3625$, or $\frac{3}{8}$, in., nearly. Hence, $d_4 = 2\frac{3}{8} + \frac{3}{4} = 3\frac{1}{8}$ in. Likewise, $d_3 = d + 2\,(\frac{1}{10}\,d + \frac{1}{8}) = 4\frac{5}{16}$ in.

To draw the taper outline of the axle, lay off half the diameter d_1 on each side of the horizontal center line of the enlarged portion n, and on either side of the vertical center line draw vertical lines at the distances indicated, assuming that the total width of this part is 12 in. Then construct the journals with their shoulders, and connect each shoulder with the adjacent diameter d_1. Finally, lay off diameter d_2 and draw the enlarged portion n. All corners should be filleted.

Solution 2.—The graphical part of this solution is shown mainly in Fig. 4 (*b*), and partly in Fig. 4 (*a*). To ascertain graphically the values of the bending moments, proceed as follows:

Produce downwards the line that indicates the position of R_2, and to any convenient scale mark off from e to f, the value of P in pounds. Choose a pole O and connect it with points e and f. Extend line $O\,e$ until it intersects at j the perpendicular representing the position of load P. From j draw a line parallel with $O\,f$, intersecting line R_1 at p. Connect p and e, and from O draw a line parallel with $p\,e$, intersecting $e\,f$ at g. The lines $f\,g$ and $g\,e$ will then represent the values of R_1 and R_2, respectively, read to the scale at which $e\,f = P$ was laid off.

The next step is to ascertain the bending moment at the enlarged portion n, as in the preceding solution. On perpendiculars dropped from any point on the axle across the polygon $p\,e\,j$, the lengths of the intercepted portions will be proportional to the bending moments at these points. The line $i\,k$, for instance, is proportional to the bending moment at the point u. To ascertain the value in inch-pounds that this line represents select an intercept whose value can easily be determined. For instance, the intercept $h\,j$ represents the bending moment $R_1\ c = 6{,}000 \times 96 = 576{,}000$ in.-lb. If the length of $h\,j$ is found to be 3.2 in., then each inch of any intercept represents $576{,}000 \div 3.2 = 180{,}000$ in.-lb. It should be observed that the larger the scale to which this polygon is drawn, the more accurate will be the readings of the intercepts.

Assuming that the strengthening action of the wheel hub may be ignored, the center of the moment is taken at q instead of at points nearer the ends of part n, which would give intercepts $b\,m$ and $i\,k$. The bending moment at q being 576,000 in.-lb., the diameter required may be found from formula **1**, Art. **4**. Thus,

$$M = \frac{S_s\, d_1^{\,3}}{10.2} = 576{,}000$$

Hence,

$$d_1 = \sqrt[3]{\frac{10.2 \times 576{,}000}{8{,}500}} = 8.84, \text{ or } 8\tfrac{7}{8}, \text{ in., nearly}$$

The remainder of the solution is wholly analytical and is identical with the corresponding part of the first solution. The dimensions d and d' having been determined by formulas based on a unit bearing pressure p of 700 lb., it may be well to check these values by calculating the diameters required to sustain the bending moments at a and a'. This may be done by measuring the intercepts at s and s', but as they are very small in the diagram, it may be more accurate to calculate the bending moments by formula **1**, Art. **4**, from which

$$d = \sqrt[3]{\frac{10.2 \times R_2 \times \frac{l}{2}}{S_s}} = \sqrt[3]{\frac{10.2 \times 12{,}000 \times 2.5625}{8{,}500}}$$

$$= 3.329, \text{ or } 3\tfrac{21}{64}, \text{ in., nearly}$$

Using the same formula,

$$d' = \sqrt[3]{\frac{10.2 \times R_1 \times \frac{l'}{2}}{S_s}} = \sqrt[3]{\frac{10.2 \times 6{,}000 \times 1.8125}{8{,}500}}$$

$$= 2.354, \text{ or } 2\tfrac{23}{64}, \text{ in., nearly}$$

As both of these values are smaller than those found by the other formula, the axle is sufficiently strong at these points to resist the bending moments. The axle may or may not be tapered as shown in Fig. 4; if it is not tapered, its diameter must be calculated from the

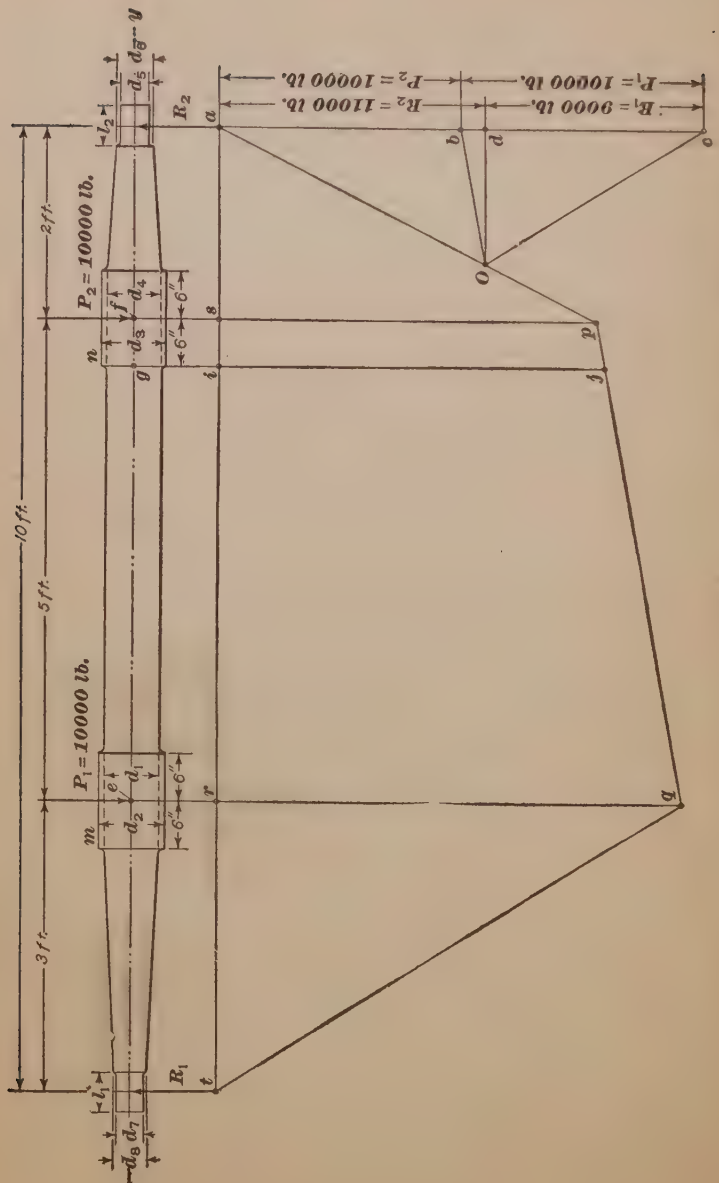
R_1 = 9000 lb.
R_2 = 11000 lb.
P_1 = 10000 lb.
P_2 = 10000 lb.
P_2 = 10000 lb.
P_1 = 10000 lb.
10 ft.
5 ft.
2 ft.
3 ft.
6"
6"
6"
6"
a
b
d
o
O
s
p
n
g
i
j
m
r
q
t
R_2
R_1
d_5
d_6
d_4
d_3
d_2
d_1
d_7
d_8
l_2
l_1
e
f
y
z

FIG. 5

maximum bending moment (hj in this case). A straight shaft is easier to construct, but the end journals, of course, must be larger, and there is considerable loss by friction on that account.

17. Axles Supporting More Than One Load.—The dimensions of axles supporting more than one load may be found graphically by the method used in Art. **16.** To assist in applying the principles, however, the following example is given:

EXAMPLE.—An axle 10 feet long between centers of bearings carries two wheels, each supporting a load of 5 tons. The distances of these loads from the ends of the axle are indicated in Fig. 5. The axle is made of steel and is not subjected to torsion. Find its dimensions.

SOLUTION.—Draw a line xy representing the axis of the axle and lay off on it, to scale, its length and the places where the loads P_1 and P_2 are applied; also the lengths of the enlarged parts m and n and the locations and directions of the reactions R_1 and R_2. On the line R_2, produced, and to any convenient scale, mark off from a to b the value in pounds of P_2, and from b to c that of P_1. Select a pole O, and connect it with points a, b, and c. Produce $a\,O$ until it intersects the produced perpendicular P_2 at p; then, from this point draw a line parallel to $O\,b$, intersecting the produced perpendicular P_1 at q. Parallel to $O\,c$, draw a line from q until it intersects R_1 at t. A line from t to a completes the polygon, and a line from O parallel to $t\,a$, intersecting $a\,c$ at d, divides the latter line into the parts $c\,d$ and $d\,a$, thus indicating the values of the reactions R_1 and R_2. By reading to the same scale as the loads P_1 and P_2, it is found that the values of R_1 and R_2 are 9,000 lb. and 11,000 lb., respectively.

Having found the reactions, ascertain the values of the intercepts in the moment diagram $a\,p\,q\,t$ by finding, for instance, the value of the intercept $q\,r$, which represents the moment $R_1 \times 3$ ft. = 9,000 lb. $\times$ 36 in. = 324,000 in.-lb. The value of the unit length of the intercepts in inch-pounds may now be found, as explained in Art. **16.**

Disregarding the supporting action of the wheel hub at m, the diameter of this part is calculated from the bending moment at e. Assuming a safe fiber stress S_4 = 9,000 lb. and using formula **1**, Art. **4**,

$$d_1 = \sqrt[3]{\frac{10.2\,M}{S_4}} = \sqrt[3]{\frac{10.2 \times 324{,}000}{9{,}000}} = 7.161, \text{ or } 7\tfrac{3}{16}, \text{ in., nearly}$$

The depth of the sunk key is $\frac{1}{6}\,d_1 = 1\frac{1}{4}$ in., nearly. The depth of the keyway is therefore $\frac{5}{8}$ in., and the diameter $d_2 = 7\frac{3}{16} + 1\frac{1}{4} = 8\frac{7}{16}$, or $8\frac{1}{2}$, in., nearly.

Proceeding to the part n, it is found that the intercept $p\,s$ at the point f is smaller than the intercept $i\,j$ at g. The latter should therefore be used as a basis for calculating the diameter at this place.

Measuring the intercept $i\,j$ to the same scale as $g\,r$, it is found to represent a bending moment $M = 270{,}000$ in.-lb. From formula **1**, Art. **4**,

$$d_3 = \sqrt[3]{\frac{10.2\,M}{S_4}} = \sqrt[3]{\frac{10.2 \times 270{,}000}{9{,}000}} = 6.74, \text{ or } 6\tfrac{3}{4}, \text{ in., nearly}$$

The increase in diameter required for the key is $\frac{1}{6}\,d_3 = 1\frac{1}{8}$ in., making $d_4 = 6\frac{3}{4} + 1\frac{1}{8} = 7\frac{7}{8}$ in., at least. The length and diameter of each journal is found again according to formulas given in Art. **15**. Assuming that $p = 700$,

$$d_5 = 1.5\sqrt{\frac{R_2}{\sqrt{S_4}\,p}} = 1.5\sqrt{\frac{11{,}000}{\sqrt{9{,}000} \times 700}} = 3.14, \text{ or } 3\tfrac{3}{16}, \text{in., nearly}$$

and the length is

$$l_2 = \frac{R_2}{p\,d_5} = \frac{11{,}000}{700 \times 3\frac{3}{16}} = 4.93, \text{ or } 5, \text{ in., nearly}$$

The height of the shoulder at this journal is

$$e = \frac{d_5}{10} + \tfrac{1}{8} = .444, \text{ or } \tfrac{1}{2}, \text{ in., nearly}$$

Hence, $d_6 = 3\frac{3}{16} + 1 = 4\frac{3}{16}$ in. Proceeding in the same manner, it is found that

$$d_7 = 1.5\sqrt{\frac{9{,}000}{\sqrt{9{,}000} \times 700}} = 2.84, \text{ or } 2\tfrac{7}{8}, \text{ in., nearly}$$

The height of the shoulder at this end is

$$e = \frac{2\frac{7}{8}}{10} + \tfrac{1}{8} = .412, \text{ or } \tfrac{7}{16}, \text{ in., nearly}$$

The diameter d_8 is then $2\frac{7}{8} + \frac{7}{8} = 3\frac{3}{4}$ in., and the length is

$$l_1 = \frac{R_1}{p\,d_7} = \frac{9{,}000}{700 \times 2\frac{7}{8}} = 4.47, \text{ or } 4\tfrac{1}{2}, \text{ in., nearly}$$

The axle is drawn in the same manner as described in Art. **16**.

SHAFTS SUBJECTED TO TORSION AND BENDING

18. Equivalent Twisting Moment.—When a shaft is subjected to combined bending and twisting moments, the two moments may, by means of a formula derived from higher mathematics, be combined in such a manner as to give an increased bending moment, which in its effect is equivalent to the combined effects of the two separate moments; this is termed the **equivalent bending moment.** Or, another formula may be used in which the two moments are so combined as to give an increased twisting moment, equal in its effect to the two separate moments; this is called the **equivalent twisting moment.** The latter method is the

more general and is the one mentioned in *Strength of Materials*, Part 2.

Let M = bending moment for any section;
T = twisting moment for the same section;
T_1 = equivalent twisting moment.

Then, $$T_1 = M + \sqrt{M^2 + T^2}$$

19. Graphical Determination of the Equivalent Twisting Moment.—It is evident that the quantity $\sqrt{M^2 + T^2}$ found in the formula of Art. **18** may represent the length of the hypotenuse in a right triangle with sides M and T. Using this length as a basis, the whole value of the formula can easily be found graphically in the following manner:

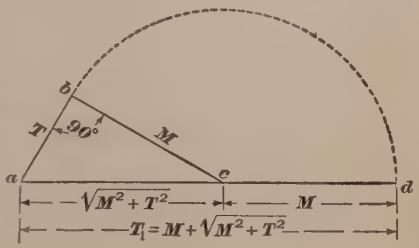

Fig. 6

Lay off the values of the moments T and M in inch-pounds, as sides $a\,b$ and $b\,c$ in the right triangle $a\,b\,c$, Fig. 6, which may be drawn to any convenient scale. The hypotenuse $a\,c$ will represent the value $\sqrt{M^2 + T^2}$, read to the same scale as the separate moments represented by the sides $a\,b$ and $b\,c$. Produce the side $a\,c$, and, in the manner indicated, lay off a length $c\,d$ equal to $b\,c$. The line $a\,d$ will then be equal to $M + \sqrt{M^2 + T^2}$, and will thus represent the total value of T_1 in inch-pounds.

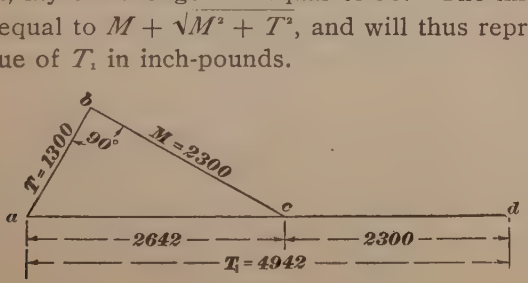

Fig. 7

Example 1.—A shaft is subjected to a bending moment of 2,300 inch-pounds and a twisting moment of 1,300 inch-pounds. Find the equivalent twisting moment.

SOLUTION.—Lay off $a\,b$ and $b\,c$, Fig. 7, at right angles to each other and to any scale, representing 1,300 and 2,300 in.-lb., respectively. Draw $a\,c$ and produce it until $c\,d$ is equal to $b\,c$. T_1 is then equal to $2{,}642 + 2{,}300 = 4{,}942$ in.-lb. Ans.

NOTE.—It is not absolutely necessary to extend the line $a\,c$, as the sum of the values represented by the lines $b\,c$ and $a\,c$ can be found by ordinary addition.

EXAMPLE 2.—Calculate the diameter of a steel crank-shaft for an engine from the following data: The length of stroke is 10 feet; the maximum tangential force acting on the crank-pin is 25 tons; and the shaft is 10 feet long between centers of bearings, and carries midway between bearings a flywheel weighing 55 tons. Assume the safe shearing stress to be 9,000 pounds $= 4\frac{1}{2}$ tons.

SOLUTION.—The twisting moment T = maximum tangential force $\times$ length of crank $= 25$ T. $\times 5$ ft. $= 125$ ft.-T. $= 1{,}500$ in.-T. The bending moment is

$$\tfrac{1}{4}\,P\,l = \frac{55 \times 10}{4} = 137.5 \text{ ft.-T.} = 1{,}650 \text{ in.-T.}$$

The value of T_1 may be determined graphically in the manner indicated in Fig. 8, but care should be taken to read off the value of T_1 in *inch-tons*, as this term is used to express the other moments. Thus, the value of T_1 is found to be 3,880 in.-T. The bending moment has therefore the same effect as if the tangential force at the crank were increased to

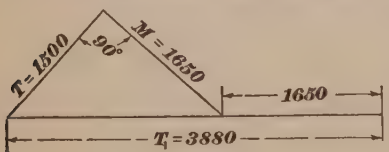

FIG. 8

$$\frac{3{,}880}{5 \times 12} = 64.7 \text{ T.}$$

If T_1 is to be found by calculation, then from the formula of Art. **18**,

$$T_1 = M + \sqrt{M^2 + T^2}$$
$$= 1{,}650 + \sqrt{1{,}650^2 + 1{,}500^2} = 3{,}880 \text{ in.-T.}$$

Now, using formula **3**, Art. **4**,

$$d = 1.72\sqrt[3]{\frac{P\,R}{S_s}} = 1.72\sqrt[3]{\frac{T_1}{4.5}} = 1.72\sqrt[3]{\frac{3{,}880}{4.5}} = 16\tfrac{3}{8} \text{ in., nearly.} \quad \text{Ans.}$$

NOTE.—In the foregoing solution it will be noticed that the units selected are inches and tons. Generally, when the bending and twisting moments are large, it is more convenient to use tons than pounds. Care must be taken, however, to make the units consistent; for example, it would be wrong to take the moments in *inch-tons*, and the safe stress in *pounds*.

EXAMPLE 3.—A wrought-iron shaft transmits 150 horsepower at 125 revolutions per minute, and at the same time is subjected to a bending moment of 60,000 inch-pounds. Calculate the diameter of the shaft.

SOLUTION.—From the formula of Art. **4**, $P\,R = 63{,}025\,\dfrac{H}{N}$; the twisting moment $T = 63{,}025 \times \frac{150}{125} = 75{,}630$ in.-lb. M being 60,000

in.-lb., T_1 is found graphically in the manner described, its value being 156,540 in.-lb. Using formula **3**, Art. **4**, viz., $d = 1.72\sqrt[3]{\frac{PR}{S_s}}$, and making $S_s = 6{,}800$,

$$d = 1.72\sqrt[3]{\frac{156{,}540}{6{,}800}} = 4\tfrac{15}{16} \text{ in., nearly.} \quad \text{Ans.}$$

20. Equivalent Torsional Moment of a Hollow Shaft.—The methods described in Arts. **18** and **19** for finding the equivalent torsional moment for solid shafts apply also to hollow shafts; but, after the moment T_1 has been determined, the diameters of the shaft must of course be found by means of formula **2**, Art. **7**.

EXAMPLE.—A hollow steel shaft transmitting 200 horsepower at 120 revolutions per minute is subjected to a bending stress of 50,000 inch-pounds. Calculate the outside and inside diameters d_1 and d_2, respectively, when the ratio $\frac{d_2}{d_1} = n = .5$.

SOLUTION.—The twisting moment T is found from the formula of Art. **4**. Thus,

$$PR = 63{,}025\frac{H}{N} = 63{,}025 \times \frac{200}{120} = 105{,}042 \text{ in.-lb.}$$

To find the equivalent twisting moment T_1 by calculation, use the formula of Art. **18**, from which $T_1 = M + \sqrt{M^2 + T^2} = 50{,}000 + \sqrt{50{,}000^2 + 105{,}042^2} = 166{,}335$ in.-lb. Inserting this value in formula **2**, Art. **7**,

$$d_1 = 1.72\sqrt[3]{\frac{PR}{S_s(1 - n^4)}} = 1.72\sqrt[3]{\frac{166{,}335}{9{,}000\ (1 - .5^4)}} = 4.646,$$

or $4\frac{11}{16}$, in., nearly

The inside diameter d_2 is $n \times d_1 = .5 \times 4.646 = 2.323$, or $2\frac{5}{16}$, in., nearly. Ans.

SHAFT COUPLINGS

21. Couplings are used to connect the ends of shafts, and are of three kinds: (1) *Fast*, or *permanent*, *couplings;* (2) *loose couplings*, or *clutches*, by means of which shafts may be connected or disconnected at pleasure; and (3) *friction clutches*, which are loose couplings that hold by friction.

PERMANENT COUPLINGS

22. Sleeve, or Muff, Couplings.—The *sleeve*, or *muff*, *coupling*, as shown in Fig. 9, consists of a cast-iron cylinder fitting over the ends of the shafts. The two ends are prevented from moving relatively to each other by a sunk key, the keyway being cut half into the box and half into the shaft ends. Quite commonly, the ends of the shafts are

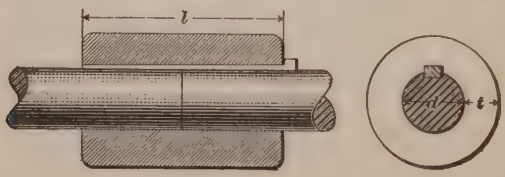

FIG. 9

enlarged so as to allow the keyway to be cut without weakening the shafts.

The key may be proportioned by the formulas already given in *Machine Design*, Part 2. For the other dimensions take

$$l = 2\tfrac{1}{2}\,d + 2 \text{ in.}$$
$$t = .4\ d + .5 \text{ in.}$$

EXAMPLE.—Find the dimensions of a muff coupling for a shaft $2\frac{1}{2}$ inches in diameter.

SOLUTION.—For the key, apply the formula given in *Machine Design*, Part 2.

$$\left.\begin{aligned} b &= \tfrac{1}{4}\,d = \tfrac{1}{4} \times 2\tfrac{1}{2} = \tfrac{5}{8} \text{ in.} \\ t &= \tfrac{1}{6}\,d = \tfrac{1}{6} \times 2\tfrac{1}{2} = \tfrac{7}{16} \text{ in.} \end{aligned}\right\} \text{Ans.}$$

For the muff, use the foregoing formula.

$$\left.\begin{aligned} l &= 2\tfrac{1}{2}\,d + 2 = 2\tfrac{1}{2} \times 2\tfrac{1}{2} + 2 = 8\tfrac{1}{4} \text{ in.} \\ t &= .4\ d + .5 = .4 \times 2\tfrac{1}{2} + .5 = 1\tfrac{1}{2} \text{ in.} \end{aligned}\right\} \text{Ans.}$$

23. Clamp Couplings.—A **clamp coupling** is shown in Fig. 10. The faces for the joint are first planed off, the holes are drilled, and then the two halves are bolted together with pieces of paper between them, after which the coupling is bored out to the exact size of the shaft. The pieces of paper on being removed leave a slight space between the halves, and the coupling when bolted to the shaft grips

it firmly. This form of coupling is very easily put on, or removed, and has no projecting parts. The key is straight, and fits only at the sides.

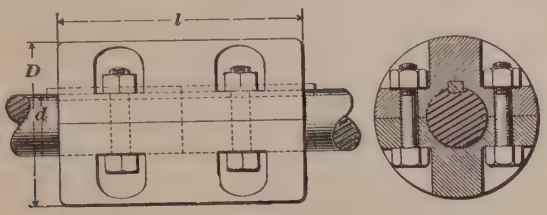

FIG. 10

The following are the proportions used in practice:

d = diameter of shaft;

D = diameter of coupling = $2\frac{1}{2}\,d + \frac{1}{2}$ in.;

l = length of coupling = $4\,d$.

The diameter of the bolts may be $\frac{5}{8}$ inch for shafts under $2\frac{1}{2}$ inches in diameter; $\frac{3}{4}$ inch for $2\frac{1}{2}$-inch shafts, and $\frac{7}{8}$ inch for larger shafts. For shafts up to 3 inches in diameter, use four bolts; for larger shafts use six bolts.

24. Flange Couplings.—A **flange coupling** is shown in Fig. 11. Cast-iron flanges are keyed to the ends of the

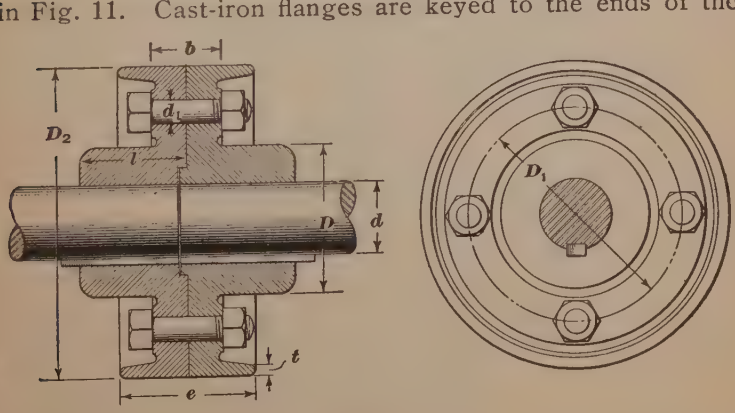

FIG. 11

shafts. To insure a perfect joint, the flange is usually faced in a lathe after being keyed to the shaft. The two flanges are then brought face to face and bolted together.

Sometimes, the ends of the shafts are enlarged to allow for the keyway. To prevent the possibility of the shafts getting out of line, one flange may be made with a projection that fits into a recess in the other, as shown.

The following proportions may be used for the form of flange coupling shown in Fig. 11:

d = diameter of shaft;

$D = 1\frac{3}{4}\,d + 1$ in.;

D_1 = diameter of bolt circle $= 2\frac{1}{2}\,d + 2$ in.;

$l = 1\frac{1}{8}\,d + 1$ in.;

n = number of bolts $= 3 + \frac{d}{2}$;

d_1 = diameter of bolts $= \frac{d}{n} + \frac{1}{4}$ in.;

D_2 = diameter of coupling $= 1.4\,D_1$;

$b = \frac{1}{2}\,d + \frac{5}{8}$ in.;

$e = 2\,b$;

$t = \frac{1}{8}\,d$.

As most designers prefer an even number of bolts, select for n the nearest even number.

The key may be proportioned by the formulas already given for keys. The dimensions for l and D_2, as found by the preceding formulas, should not be used for shafts exceeding 4 inches in diameter. For larger shafts, the dimensions given in Table II may be used.

TABLE II

DIMENSIONS OF COUPLINGS FOR SHAFTS OVER FOUR INCHES IN DIAMETER

Diameter of Shaft Inches	l Inches	D_2 Inches	Diameter of Shaft Inches	l Inches	D_2 Inches
$4\frac{7}{16}$	$6\frac{1}{4}$	$14\frac{1}{2}$	7	10	22
5	$7\frac{1}{4}$	$16\frac{1}{2}$	8	11	24
$5\frac{1}{2}$	8	$17\frac{1}{2}$	9	12	26
6	$8\frac{1}{2}$	20	10	13	28
$6\frac{1}{2}$	$9\frac{1}{4}$	21			

25. The Sellers Cone Coupling.—The **Sellers cone coupling,** which is shown in Fig. 12, consists of an outer box, or muff, that is cylindrical externally, but has the form of a double truncated cone on the inside. Within the muff are placed two slotted sleeves, which are turned on the outside to fit the muff and are also bored out to fit the shaft. These sleeves are pulled together by three bolts, and as the former are drawn farther into the muff, they grip it and the shaft firmly.

The bolt holes pass through both the sleeves and the muff, and are square in cross-section. The friction between the

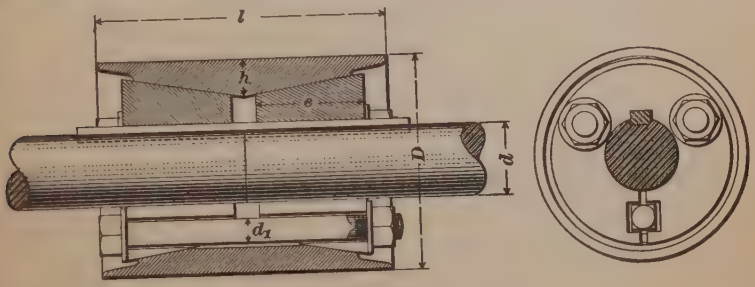

FIG. 12

sleeves and the shaft is generally sufficient to prevent slipping, but to be on the safe side, the sleeves are usually keyed to the shaft. The key should have no taper, and should fit at the sides only; its proportions may be obtained by formulas given in *Machine Design*, Part 2. The other proportions may be taken as follows:

d = diameter of shaft; $\quad d_1 = \frac{1}{3}\,d$;

$l = 4\,d$; $\quad e = 1\frac{1}{2}\,d$;

$D = 3\,d$; $\quad h = \frac{1}{2}\,d$.

The conical sleeves may be tapered 4 inches per foot of length. In putting up lines of shafting, the couplings, if possible, should be placed near bearings.

LOOSE COUPLINGS

26. Claw Couplings.—**Claw couplings** are used when the shafts are to be alternately connected and disconnected. For large slow-moving shafts, the claw coupling shown in

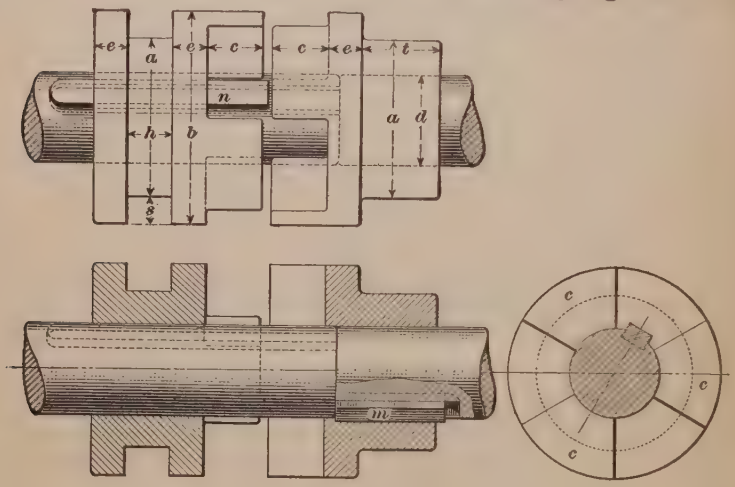

FIG. 13

Fig. 13 may be used. This coupling somewhat resembles the flange coupling shown in Fig. 11, except that the flanges, instead of being bolted together, are provided with a set of lugs *c* that interlock. One flange is permanently fastened to the shaft by a sunk key *m*, while the other is fastened to its shaft by a feather key *n*, and may be moved back and forth, thus throwing the coupling, or clutch, in or out of gear. The movement of the clutch is effected by a forked lever fitting into the recess *h*.

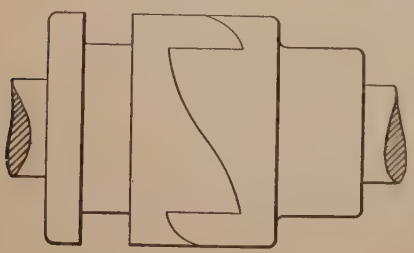

FIG. 14

The lugs, or claws, may be given the form shown in Fig. 14, in which case the couplings are more easily put in gear, but will drive in only one direction.

Cast-iron claw couplings may have the following proportions:

d = diameter of shaft;	$e = \frac{3}{8}\,d$;
$a = 1\frac{3}{4}\,d$;	$h = \frac{1}{2}\,d$;
$b = 2\frac{3}{8}\,d$;	$s = \frac{5}{16}\,d$;
$c = \frac{5}{8}\,d$;	$t = \frac{7}{8}\,d$.

FRICTION CLUTCHES

27. Function of the Friction Clutch.—When it is desired to connect two machine parts—one revolving and the other stationary—without injurious shock to the machinery, the motion must be imparted gradually to the driven part. This gradual starting, however, is impossible without some relative motion between the two parts during the starting period. In a friction clutch, this relative motion can readily be provided, because motion is transmitted from one part to the other solely by means of the friction of the contact surfaces. A gradual increase of this friction is followed by a reduction of the relative motion until both parts revolve at the same speed.

In order to produce relative motion between two bodies in contact, a certain force P is necessary to overcome the friction F between the rubbing surfaces. The amount of this force P depends on the nature of the contact surfaces and on the pressure N normal to them. The ratio between the frictional force F and the normal N, termed *the coefficient of friction*, is generally designated by f; hence,

$$f = \frac{F}{N} \qquad (1)$$

It is also known that if e is the angle of friction, then

$$f = \tan e = \frac{F}{N} \qquad (2)$$

28. For contact between metals, and between metals and wood, f varies between .1 and .5. For cast iron on cast iron it varies from .1 to .15.

From formula **1**, Art. **27**, $F = f\,N$. As f is always less than **1**, it follows that F must always be smaller than N.

For instance, for cast iron on cast iron, $F = fN = .1N$. If the pressure N is 100 pounds, then the resistance F will be $.1 \times 100 = 10$ pounds.

When frictional forces are used to transmit large powers, it is evident that the normal pressure N must be very large. As it is undesirable to produce this directly, a friction clutch must be so designed that only a relatively small pressure will be required to bring the clutch into action so as to produce the necessary friction. This may be attained by different means, one of which is to make the rubbing surfaces parts of cones, depending on their wedge action to produce the pressure required.

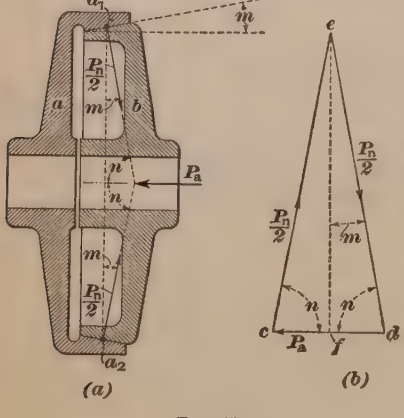

FIG. 15

29. Cone Friction Clutch.—Fig. 15 (*a*) illustrates diagrammatically the two cones of a **cone friction clutch**, *a* being the external and *b* the internal one. The two cones are assumed to be held in contact by means of the axial force P_a, and its components $\frac{P_n}{2}$ exert pressures normal to the contact surfaces of the cones. The angle m indicates the inclination of the cone surface toward its axis, and the angle n is the complement of this angle. A uniform pressure is supposed to be acting normally over the whole circular surface of each cone, but, to facilitate the explanation of its action, this pressure is shown concentrated at the two points a_1 and a_2 located diametrically opposite each other, each component being equal to $\frac{P_n}{2}$.

The magnitude and direction of these components may be found by constructing a triangle as shown in Fig. 15 (*b*), in which the side $c\,d$, drawn to any suitable scale, represents

the direction and value of the force P_a. By drawing the lines ce and de at angles n to cd and letting them intersect at e, the directions and values of the two forces $\frac{P_n}{2}$ are found. When a perpendicular is drawn from e to cd, then $df = \frac{P_a}{2}$, and $\frac{P_a}{2} = \sin m \frac{P_n}{2}$. Hence, $P_a = \sin m \, P_n$, and

$$P_n = \frac{P_a}{\sin m} \qquad (1)$$

When the cone a revolves, the friction produced at the contact surfaces of cones a and b acts as a tangential force P_t, not shown in the illustration, which tends to rotate the cone b. One-half of this force, that is, $\frac{P_t}{2}$, is supposed to be concentrated at each of the two points a_1 and a_2. Inserting the letters P_t and P_n for F and N, respectively, in formula **1**, Art. **27**, then $f = \frac{P_t}{P_n}$, and

$$P_t = f P_n \qquad (2)$$

The twisting moment T tending to turn the cone b is $R \times 2\frac{P_t}{2} = R P_t$, in which R = distance of points a_1 or a_2 from the axis of the cone b. From formula **2** the tangential force $P_t = f P_n$, and from formula **1**, $P_n = \frac{P_a}{\sin m}$. Hence,

$$P_t = f P_n = f \frac{P_a}{\sin m}$$

and

$$T = R P_t = \frac{f P_a R}{\sin m} \qquad (3)$$

If in place of the moment T it is desired to express the action of the clutch in horsepower, then from the formula $T = PR = 63{,}025 \frac{H}{N}$,

$$T = R P_t = \frac{f P_a R}{\sin m} = \frac{63{,}025 \, H}{N}$$

or,

$$H = \frac{f P_a R N}{63{,}025 \sin m} \qquad (4)$$

Let A = frictional area of a cone or cylinder, in square inches;

p_s = safe pressure per square inch between the surfaces;

w = width of conical or cylindrical surface;

R = radius of a cylindrical surface, and average radius of a conical surface.

Then, $$A = \frac{P_n}{p_s} \qquad (5)$$

As $A = 2\pi R w$,

$$w = \frac{A}{2\pi R} \qquad (6)$$

An average value for p_s is 50 pounds per square inch. Referring to Fig. 16, the average diameter of the conical

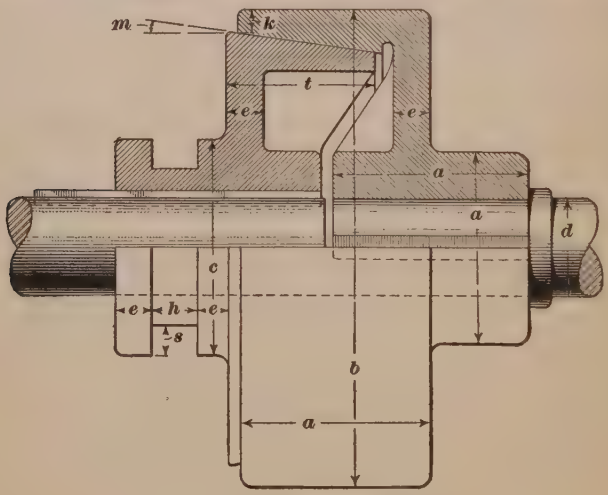

FIG. 16

part may be from four to eight times the shaft diameter, according to the amount of power to be transmitted. The angle m of the cone may be from 4° to 10°. The other proportions are as follows:

d = diameter of shaft;

$a = 2\ d$;	$e = \frac{3}{8}\ d$;
$b = 4$ to $8\ d$;	$h = \frac{1}{2}\ d$;
$c = 2\frac{1}{4}\ d$;	$s = \frac{5}{16}\ d$;
$t = 1\frac{1}{2}\ d$;	$k = \frac{1}{4}\ d$.

30. The following example will illustrate the application of the formulas for the cone friction clutch.

EXAMPLE.—Ascertain the width of the cone surfaces in a cast-iron friction clutch intended to transmit 20 horsepower at 200 revolutions per minute, when the mean radius of the cone surface is 5 inches and the pressure per square inch of same is not to exceed 55 pounds.

SOLUTION.—Substituting P_t for P in the formula $PR = 63{,}025\frac{H}{N}$,

$$P_t = 63{,}025\frac{H}{RN} = 63{,}025 \times \frac{20}{5 \times 200} = 1{,}260 \text{ lb., nearly}$$

As both cones are made of cast iron, the value of f may be taken as .15. According to the laws of friction, if the angle m is made equal to or a little greater than the friction angle e for cast iron, the two friction surfaces will separate voluntarily when released. As $\tan e = f = .15$, $e = 8.5°$, nearly. Transposing formula **2**, Art. **29**, and solving for P_n, then,

$$P_n = \frac{P_t}{f} = \frac{1{,}260}{.15} = 8{,}400 \text{ lb.}$$

In case f should occasionally assume a greater value than .15, it is well to increase the angle m to prevent the cones from sticking together when released. Let the value m be increased to 11°, corresponding to a value $f = .194$. Then, the axial pressure P_a necessary to produce the pressure P_n is found from formula **1**, Art. **29**, from which

$$P_a = P_n \sin m = 8{,}400 \times .191 = 1{,}604 \text{ lb., nearly}$$

From formula **5**, Art. **29**,

$$A = \frac{8{,}400}{55} = 153 \text{ sq. in., nearly}$$

From formula **6**, Art. **29**,

$$w = \frac{153}{2 \times 3.1416 \times 5} = 4.87, \text{ or } 4\tfrac{7}{8}, \text{ in., nearly. Ans.}$$

31. Cylinder Friction Clutch.—When the rubbing surfaces of a clutch are cylindrical, formulas **2, 5,** and **6,** Art. **29,** may be applied, but not formulas **3** and **4.** Eliminating the value sin m, the ratio of P_a to P_n will depend on the mechanism employed for transforming the axial force P_a into the radial forces $\frac{P_n}{2}$.

An example of a **cylinder friction clutch** is shown in Fig. 17. The shaft n carries a flange, or cylinder, A, and the shaft m has keyed to it a ring B. The ring is split and fits inside the flange, or cylinder, A, the split ends being

connected by a screw having a right- and a left-hand thread. The screw is turned by the lever C, which is connected by a link D to the sleeve E. When the sleeve is pushed toward the clutch, the rotation of the screw throws the ends F, F of the ring B apart and thereby causes the ring B to grip the

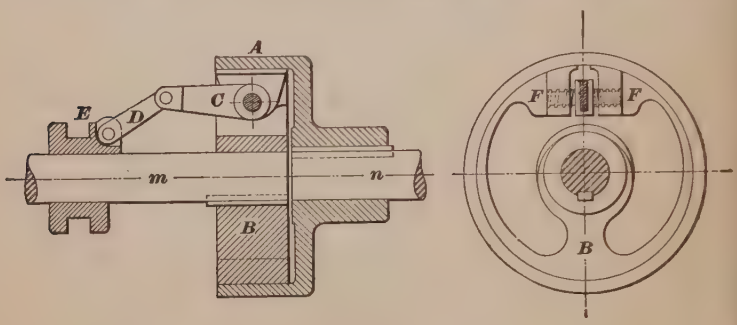

FIG. 17

flange A tightly. A clutch of this form is easy to operate and produces no end thrust on the shaft.

Views showing details of the ends of the ring B, Fig. 17, are given in Fig. 18, (a) being an elevation and (b) a section taken on the line $x\,x$. To facilitate the insertion of the

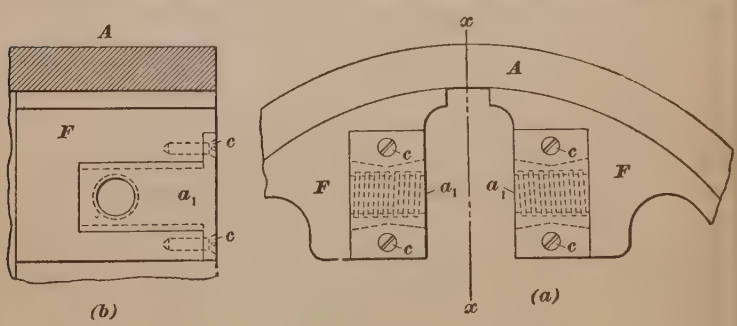

FIG. 18

screw shown in Fig. 17, it is necessary to make the nuts a_1, a_1 separate from the ring F. The nuts are dovetailed into recesses in the ring, as indicated. After the nuts have been screwed into position on the screw, they are inserted in the recesses and held there by the screws c, c.

Another form of clutch differs from the preceding in having the friction band B on the outside of the flange. The principal proportions of these clutches are about the same as those given for Fig. 16.

32. Disk Friction Clutches.—In order to secure the required frictional surface when conditions limit the diameter of the clutch, it is necessary to extend the dimensions of the clutch lengthwise and to have the friction surfaces at right angles to the shaft. An example of this class is the **disk friction coupling.**

The construction of this clutch when used for coupling a spur gear to a shaft is shown in Fig. 19. The spur gear g,

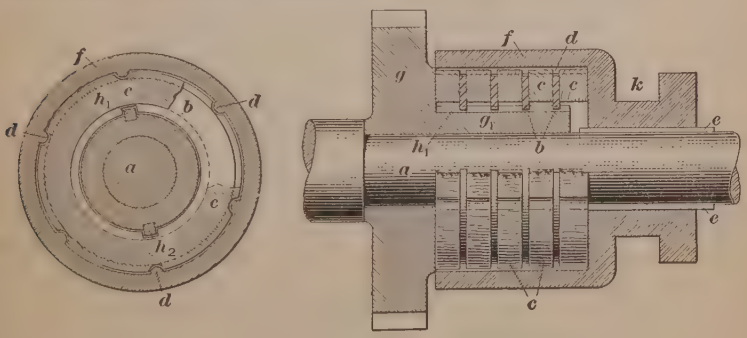

FIG. 19

which is provided with a hub g_1, revolves loosely on the shaft a, and on hub g_1 are feathers h_1 and h_2 that engage with wrought-iron rings b, b. The coupling box f is connected to the shaft by means of feathers e, e, which permit the box to be moved lengthwise. The box is provided with six longitudinal ribs d, d that engage with grooves in the wooden rings c, c, and consequently these rings will revolve with the box and the shaft, while the gear g and the rings b, b remain stationary. By means of a forked lever resting in the groove k, the box may be forced to the left, bringing all the rings in contact and compelling the rings b and the gear g to revolve with the shaft.

Let R = mean radius of the rubbing surfaces;

n = number of pairs of rubbing surfaces.

Then, formulas 3 and 4, Art. 29, will assume the following forms:

$$T = RP_t = nfP_aR \qquad (1)$$

and

$$H = \frac{nfP_aRN}{63{,}025} \qquad (2)$$

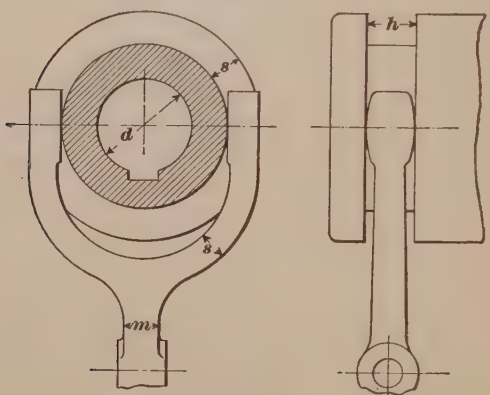

FIG. 20

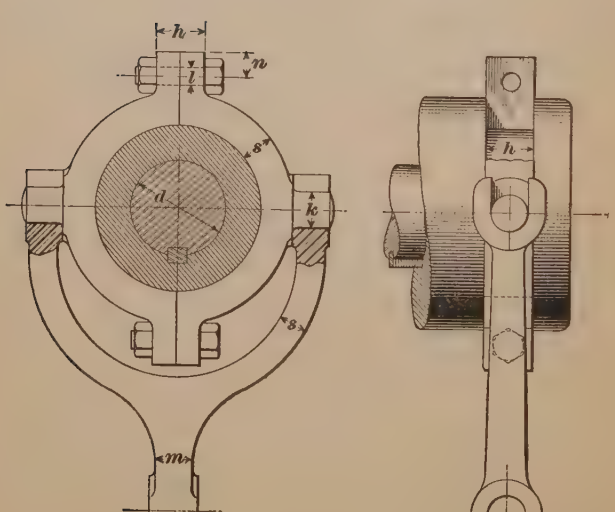

FIG. 21

33. Shifting Gear for Clutches.—A clutch is usually put in or out of gear by means of a **forked lever,** the

prongs of which fit into the groove cut in the sliding part of the clutch. The lever is usually operated by hand, though for large clutches the end of the lever may be moved by a screw and hand wheel. The ordinary design of the forked end of the lever is shown in Fig. 20. To increase the wearing surface, however, a strap as shown in Fig. 21 may be used. The strap completely fills the groove, and is often made of brass. The dimensions h and s are the width and the depth of the groove, respectively, which are to be determined by the rules already given for the proportions of the clutch.

The proportions for the clutch lever are:

d = diameter of shaft; $s = \frac{5}{16}d$;
$h = \frac{1}{2}d$; $m = \frac{3}{8}d$;
$k = \frac{3}{8}d$; $n = \frac{1}{4}d$.
$l = \frac{3}{16}d$;

SPRINGS

GENERAL FUNCTION OF SPRINGS

34. Most machine parts are constructed so as to show a minimum amount of deflection, but some are designed to give a maximum amount of deflection for a given stress. Such parts are termed **springs,** and their function is either that of providing a yielding connection between two other machine parts, or of providing means for graduating the pressure or pull on a certain machine part in accordance with a certain fixed value. An example of the first kind are the springs under a railroad car, while the springs in safety valves, governors, etc. are examples of the second kind.

Springs are divided into three general classes: *elliptical*, *helical*, and *spiral*.

ELLIPTICAL SPRINGS

35. Plate, or Cantilever, Spring.—The **plate spring,** whether supported at one end or at the center, is considered as a cantilever, and the formulas for such springs are based on the formulas for cantilevers. In the case of a plate spring supported at the center, one-half of the load is

supposed to be carried at each end of the spring; each half of the spring is thus treated as a cantilever supporting one-half the load.

It was shown in *Strength of Materials*, Part 2, that the bending moment of a beam

$$M = S_s \frac{I}{c} \qquad (1)$$

if S_s = safe working stress, in pounds per square inch;
I = moment of inertia of its cross-section;
c = distance of outer fiber from neutral axis, in inches.

It was also shown that the deflection of a beam

$$s = \frac{a W l^3}{E I} \qquad (2)$$

when a = a constant depending on the mode of loading;
W = concentrated load, in pounds;
d = depth of beam, in inches;
l = length of beam, in inches;
E = coefficient of elasticity = $E_t = E_c$.

For a rectangular cross-section $I = \frac{b d^3}{12}$ and $c = \frac{d}{2}$. By inserting these values in formula **1**,

$$M = \frac{2 S_s b d^3}{12 d} = \frac{S_s b d^2}{6}$$

For a cantilever with a concentrated load at one end, $M = Wl$; hence, $Wl = \frac{S_s b d^2}{6}$, and

$$W = \frac{S_s b d^2}{6 l} \qquad (3)$$

If the same values of I and c are inserted in formula **2**, it becomes

$$s = \tfrac{1}{3} \frac{12 W l^3}{E b d^3}$$

or,

$$s = \frac{4 W l^3}{E b d^3} \qquad (4)$$

36. Variations of Deflection.—If the dimensions of a spring intended to support a given load W are calculated by means of formula **3**, Art. **35**, it may be found on applying formula **4**, Art. **35**, for the purpose of ascertaining the

deflection s resulting from the load W, that this deflection is either too large or too small. As a guide in making the necessary changes in the dimensions of the spring, so as to produce the required deflection, it should be noted that if the length l is changed, it is obvious from formulas **3** and **4**, Art. **35**, that for a given increase in length l the gain in deflection is much greater than the loss in strength. A decrease in the width b for the purpose of increasing the deflection is followed by a corresponding loss in strength. Increasing the depth d will decrease the deflection, and increase the strength of the spring, though not in the same ratio.

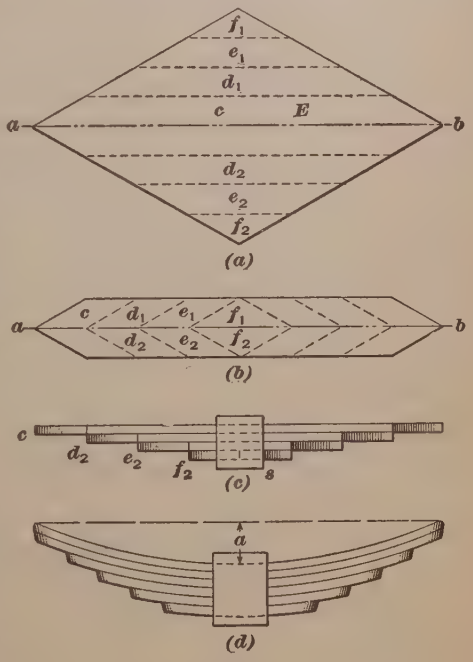

FIG. 22

37. Design of Elliptical Springs. The dimensions of a spring need not be uniform throughout its length; it may vary in thickness or in width. If the rate of increase in width is directly proportional to the bending moment, then the stress S_s will be uniform from end to end. Such a spring is shown in Fig. 22 (a), which is a plan view of a flat spring E that is supported along the middle line $f_1 f_2$ and consists of two triangular cantilevers joined end to end. For the purpose of economizing space, it is preferable to subdivide the spring into narrow strips, or *leaves*, as shown by the dotted lines, and to lay these leaves on top of one another.

In Fig. 22 (*b*), the central strip *c* is on top, and below it are the narrower strips d_1, d_2, e_1, e_2, etc., which are placed side by side and supposedly united along the center line *a b*.

Fig. 22 (*c*) shows a side view of the spring built up from the strips, which are held together by the strap *s* and form what is termed a *laminated spring*. In practice, the separate leaves are given a curved form before being inserted in the strap, and the spring then assumes the form shown in Fig. 22 (*d*). In this form, it is called a *semielliptic spring*, and if two such springs are joined back to back, it is a *full-elliptic spring*. The distance *a* is called the **set,** or **arch,** of the spring. In general, the upper leaf, whether single or double, is left with square ends, while the others have their ends rounded off.

The deflection calculated for the undivided spring *E*, Fig. 22 (*a*), will not correspond to the actual deflection of the laminated spring, Fig. 22 (*d*). In the latter form, the friction between adjoining plates prevents the deflection from reaching its full value when the spring is straightening out under an increasing load. When the load is decreasing, the friction will also prevent the spring from returning to the position it should occupy in accordance with the load, and will therefore show a deflection that is too large.

The following formulas for elliptical springs, deduced by Reuleaux, have, in general, given satisfactory results:

Let s = deflection at end of spring, in inches;
W = load at one end of spring, in pounds;
E = modulus of elasticity for bending = $E_t = E_c$;
l = one-half length of spring, in inches, when loaded;
b = width of plates, in inches;
t = thickness of one plate, in inches;
n = number of plates in spring;
S_t = safe maximum fiber stress in plate.

Then,
$$W = \frac{S_t n b t^2}{6 l} \qquad (1)$$

and
$$s = \frac{6 W l^3}{E n b t^3} \qquad (2)$$

The latter formula gives correct values for *s* if the plates gradually decrease in length, as shown in Fig. 22 (*c*); but

frequently, as shown in Fig. 22 (*d*), an extra plate of the same length as plate *c* is added. In such cases, the factor 6 of formula **2** is reduced to 5.5 and

$$s = \frac{5.5\ W l^3}{E n b t^3} \qquad (3)$$

In general, for steel, S_s is taken as 80,000 pounds and E, as 30,000,000 pounds, in which case formulas **1** and **2** assume the forms

$$W = \frac{13{,}333\ n b t^2}{l} \qquad (4)$$

and

$$s = \frac{W l^3}{5{,}000{,}000\ n b t^3} \qquad (5)$$

EXAMPLE.—An elliptical spring, similar to Fig. 22 (*d*), consists of 5 plates 3 inches wide and $\frac{3}{8}$ inch thick. The two upper plates are of equal length. The length of the spring is 40 inches and it supports a load of 1,200 pounds. Find the deflection and the safe load.

SOLUTION.—From formula **5**,

$$s = \frac{600 \times 20^3}{5{,}000{,}000 \times 5 \times 3 \times .375^3} = 1.2 \text{ in.} \quad \text{Ans.}$$

From formula **4**,

$$W = \frac{13{,}333 \times 5 \times 3 \times .375^2}{20} = 1{,}406 \text{ lb. for each end of spring.} \quad \text{Ans.}$$

HELICAL SPRINGS

38. Springs of Round Wire.—A helical spring consists of a wire or rod wound into helical coils, as shown

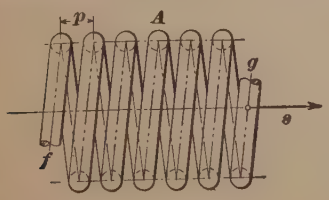

FIG. 23

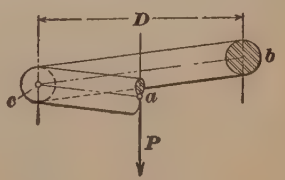

FIG. 24

in Fig. 23, in which A is the coiled wire, fg its center line, and p the pitch, or amount of advance for each turn. The coils may touch one another, or there may be a space between the successive turns, as shown.

In Fig. 24, $a\,b\,c$ represents part of one coil. If a load P is acting at the point a and the cross-sectional area at c is considered with reference to this load, the wire is subjected to a twisting action, the twisting moment T being $\frac{PD}{2}$, nearly. It is evident that the action of this twisting moment is not limited to the point c, but is also transmitted along the wire, and the effect is the same as if the coil were a straight wire or shaft subjected to simple torsion.

An end view of a spring supposed to be uncoiled into a straight piece of wire is shown at u, Fig. 25. The wire is provided with a pulley B having a diameter D equal to that of the coil in Fig. 24. The action of the load P, Fig. 25, will therefore have the same effect on the wire as the load P, Fig. 24, considered with reference to the point c of the wire. If a load P suspended from the pulley B turns the latter so that the point a_1 moves into the position a_2, then a point b_1 on the wire, while moving into the position b_2, will move through an angle e. The angular deflection of the wire is therefore $e°$. If s is the length of path through which point a_1 has moved, then

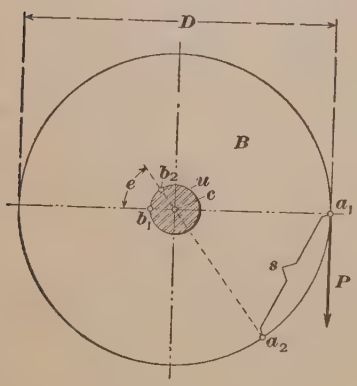

Fig. 25

$$s : \pi D = e° : 360°$$

and

$$e = \frac{360\,s}{\pi D} \qquad (1)$$

In Art. 4, it was shown that the twisting moment

$$T = \frac{S_s J}{c} \qquad (2)$$

when S_s = safe fiber stress for shear;
J = polar moment of inertia;
d = diameter of shaft;
c = distance from outer fiber to neutral axis.

For a circular section, $J = \frac{\pi d^4}{32}$ and $c = \frac{d}{2}$. Inserting these values in formula **2**,

$$T = \frac{S_s \pi d^3}{16}$$

If P is the safe load in pounds, the safe twisting moment is $\frac{PD}{2} = \frac{S_s \pi d^3}{16}$; hence,

$$P = \frac{2 S_s \pi d^3}{16 D} = \frac{S_s d^3}{2.55 D}$$

or,

$$P = .3927 \frac{S_s d^3}{D} \qquad (3)$$

39. It has been shown in Art. **5** that the angular deflection of a shaft

$$e = \frac{584 T l}{E_s d^4} \qquad (1)$$

If n is the number of coils in the spring, then the length of the uncoiled spring is $l = n \pi D$, nearly. This value is near enough for most practical purposes.

NOTE.—If the correct length is required, the pitch of the coils should be considered, as indicated by p, Fig. 23. In this case $l = n\sqrt{p^2 + (\pi D)^2}$.

Inserting the value $l = n \pi D$ in formula **1**,

$$e = \frac{584 T n \pi D}{E_s d^4}$$

Substituting for e the value given in formula **1**, Art. **38**, and for T the value $\frac{PD}{2}$,

$$\frac{360 s}{\pi D} = \frac{584 P D^3 \pi n}{2 E_s d^4}$$

and

$$s = \frac{8 P D^3 n}{E_s d^4} \qquad (2)$$

The total twist s of the wire u, as represented by the length of the curved path traced by the point a_1 on the pulley B, Fig. 25, will show itself as an axial motion in the coiled spring. This is because each coil is subjected to a twisting action in a direction nearly at right angles to the axis of the

wire. Therefore, the sum of all the resulting twisting motions will also be in a direction at right angles to all the coils; that is, in the direction of the axis of the spring. The direction of this motion for a spring in tension is shown by the arrow s, Fig. 23.

Distinction should be made between the value of P in formula **2** of this article and the value of P in formula **3**, Art. **38**. In the latter formula, P represents the safe load that the spring may support; in the former, P is the load for which the deflection is to be ascertained. The value used for P in formula **2** must never be greater than the safe load P found by formula **3**, Art. **38**.

The factor n is not contained in formula **3**, Art. **38**; consequently, the number of coils n may be varied to give a desired deflection without affecting P.

40. Compression of Springs.—The formulas given in Arts. **38** and **39** apply also to helical springs subjected to compression, since, from Fig. 25, it is immaterial whether the rod u is twisted to the left or to the right. However, the following difference between springs under tension and those under compression should be noted. In a spring under tension, the coils may be in contact while the spring is in its initial position; in a spring under compression, the pitch of the coils, while the spring is in its initial position, must be such that sufficient space is left for the coils to approach the required distance; that is, the pitch p, Fig. 23, must be larger than $\frac{s}{n}$.

EXAMPLE 1.—A helical spring made of $\frac{1}{2}$-inch round steel has a diameter D of 3 inches. (a) Calculate the load P that it will carry safely if $S_s = 60{,}000$ pounds. (b) Calculate the extension, if the spring has 15 coils and supports a load P of 700 pounds.

SOLUTION.—(a) Substituting these values in formula **3**, Art. **38**,

$$P = \frac{.3927 \times 60{,}000 \times .5^3}{3} = 982 \text{ lb.} \quad \text{Ans.}$$

(b) Use formula **2**, Art. **39**. Thus, if $E_s = 12{,}000{,}000$, then

$$s = \frac{8 \times 700 \times 3^3 \times 15}{12{,}000{,}000 \times .5^4} = 3.024, \text{ or 3, in., nearly.} \quad \text{Ans.}$$

EXAMPLE 2.—A helical spring is to be made of $\frac{7}{16}$-inch round steel wire and must show an extension of 5 inches for a load of 500 pounds. Find the diameter of the coils and the required number.

SOLUTION.—Applying formula 2, Art. 39, and solving for the product $D^3 n$,

$$D^3 n = \frac{s E_s d^4}{8 P} = \frac{5 \times 12{,}000{,}000 \times .4375^4}{8 \times 500} = 550$$

Hence, $n = \frac{550}{D^3}$. Inserting a tentative value of $D = 3$,

$$n = \frac{550}{27} = 20 \text{ coils, nearly.} \quad \text{Ans.}$$

It is necessary to ascertain whether this value of D will give the spring sufficient strength to support a load of 500 lb. By formula 3, Art. 38,

$$P = \frac{.3927 \times 60{,}000 \times .4375^3}{3} = 657.7 \text{ lb.}$$

showing that the spring is of ample strength.

41. Working Stress.—Since, owing to its elasticity, a spring is not subjected to sudden shocks, as would be the case with a rigid body under similar conditions, a greater safe working stress may be used. Hence, it is customary to take the value of S_s as one-half the ultimate shearing strength of the material. In preceding examples, the value of S_s for tempered cast steel has been taken as 60,000 pounds. This value is sufficiently safe for conditions in which the spring is required to act only intermittently, as when used in a safety valve or an engine governor, but if the spring is to be in constant action, a lower value of S_s should be used.

42. Table of Strength and Deflection.—In order that the calculations of the strength and deflection of helical springs may be simplified, Table III, computed by J. Begtrup is here inserted. This table is based on the formulas for round steel wire, the derivations of which were given in Arts. 38 and 39. The symbols used in the table differ in some respects from those employed in the preceding formulas, as will be seen from the list that follows. It should be particularly noted that S_1 represents the deflection of one coil at a load of 100 pounds. Thus, in the table,

TABLE III

STRENGTH AND DEFLECTION OF HELICAL SPRINGS OF ROUND STEEL

d = .065 in. No. 16	D_1	.25	.50	.75	1.00	1.25	1.50	1.75	2.00		
	P	35	15	9	7	5	4.5	3.8	3.3		
	s_1	.0276	.3588	1.433	3.562	7.250	12.88	20.85	31.57		
		.0236	.3075	1.288	3.053	6.214	11.04	17.87	27.06		
		.0197	.2562	1.023	2.544	5.178	9.200	14.89	22.55		
d = .120 in. No. 11	D_1	.50	.75	1.00	1.25	1.50	1.75	2.00	2 25	2.50	
	P	107	65	46	36	29	25	22	19	17	
	s_1	.0206	.0937	.2556	.5412	.9856	1.624	2.492	3.625	5.056	
		.0176	.0804	.2191	.4639	.8448	1.392	2.136	3.107	4.334	
		.0147	.0670	.182	.3866	.7040	1.160	1.780	2.589	3.612	
d = .180 in. No. 7	D_1	.75	1.00	1.25	1.50	1.75	2.00	2.25	2.50	2.75	3.00
	P	241	167	128	104	88	75	66	59	53	49
	s_1	.0137	.0408	.0907	.1703	.2866	.4466	.6571	.9249	1.256	1.660
		.0118	.0350	.0778	.1460	.2457	.3828	.5632	.7928	1.077	1.423
		.0098	.0292	.0648	.1217	.2048	.3190	.4693	.6607	.8975	1.186

$d = \frac{1}{4}$ in.	D_1	1.25	1.50	1.75	2.00	2.25	2.50	2.75	3.00	3.25	3.50	
	P	368	294	245	210	184	164	147	134	123	113	
	s_1	.0199	.0389	.0672	.1067	.1593	.2270	.3109	.4139	.5375	.6835	
		.0171	.0333	.0576	.0914	.1365	.1944	.2665	.3548	.4607	.5859	
		.0142	.0278	.0480	.0762	.1137	.1610	.2221	.2957	.3839	.4883	
$d = \frac{5}{16}$ in.	D_1	1.50	1.75	2.00	2.25	2.50	2.75	3.00	3.25	3.50	3.75	4.00
	P	605	500	426	371	329	295	267	245	226	209	195
	s_1	.0136	.0242	.0392	.0593	.0854	.1187	.1583	.2066	.2640	.3312	.4089
		.0117	.0207	.0336	.0508	.0732	.1012	.1357	.1771	.2263	.2839	.3505
		.0097	.0173	.0280	.0424	.0610	.0853	.1131	.1476	.1886	.2366	.2921
$d = \frac{3}{8}$ in.	D_1	2.00	2.25	2.50	2.75	3.00	3.25	3.50	3.75	4.00	4.25	4.25
	P	765	663	589	523	473	433	398	368	343	321	301
	s_1	.0169	.0259	.0377	.0528	.0711	.0935	.1200	.1513	.1874	.2290	.2761
		.0145	.0222	.0323	.0452	.0610	.0801	.1029	.1297	.1606	.1963	.2367
		.0120	.0185	.0269	.0376	.0508	.0668	.0858	.1081	.1338	.1635	.1972
$d = \frac{7}{16}$ in.	D_1	2.00	2.25	2.50	2.75	3.00	3.25	3.50	3.75	4.00	4.50	5.00
	P	1,263	1,089	957	853	770	702	644	596	544	486	432
	s_1	.0081	.0126	.0186	.0262	.0357	.0472	.0617	.0772	.0960	.1423	.2016
		.0069	.0108	.0160	.0225	.0306	.0405	.0529	.0661	.0823	.1220	.1728
		.0058	.0090	.0133	.0187	.0255	.0337	.0441	.0551	.0686	.1017	.1440

TABLE III—(*Continued*)

$d = \frac{1}{2}$ in.	D_1	2.00	2.25	2.50	2.75	3.00	3.25	3.50	3.75	4.00	4.50	5.00
	P	1,963	1,683	1,472	1,309	1,178	1,071	982	906	841	736	654
	s_1	.0042	.0067	.0099	.0141	.0194	.0259	.0336	.0427	.0534	.0796	.1134
		.0036	.0057	.0085	.0121	.0167	.0222	.0288	.0366	.0457	.0683	.0972
		.0030	.0048	.0071	.0101	.0139	.0185	.0240	.0305	.0381	.0569	.0810
$d = \frac{9}{16}$ in.	D_1	2.50	2.75	3.00	3.25	3.50	3.75	4.00	4.25	4.50	5.00	5.50
	P	2,163	1,916	1,720	1,560	1,427	1,315	1,220	1,137	1,065	945	849
	s_1	.0056	.0081	.0112	.0151	.0197	.0252	.0316	.0390	.0474	.0679	.0935
		.0048	.0070	.0096	.0129	.0169	.0216	.0271	.0334	.0406	.0582	.0801
		.0040	.0058	.0080	.0108	.0141	.0180	.0225	.0278	.0339	.0485	.0668
$d = \frac{5}{8}$ in.	D_1	2.50	2.75	3.00	3.25	3.50	3.75	4.00	4.25	4.50	5.00	5.50
	P	3,068	2,707	2,422	2,191	2,001	1,841	1,704	1,587	1,484	1,315	1,180
	s_1	.0034	.0049	.0068	.0092	.0121	.0155	.0196	.0243	.0297	.0427	.0591
		.0029	.0042	.0058	.0079	.0104	.0133	.0168	.0208	.0254	.0366	.0506
		.0024	.0035	.0049	.0066	.0086	0111	.0140	.0173	.0212	.0305	.0422

$d = \frac{11}{16}$ in.	D_1	3.00	3.25	3.50	3.75	4.00	4.25	4.50	4.75	5.00	5.50	6.00
	P	3.311	2.988	2.723	2.500	2.311	2.151	2.009	1.885	1.776	1.591	1.441
	s_1	.0043	.0058	.0077	.0100	.0127	.0157	.0193	.0233	.0279	.0388	.0522
		.0037	.0050	.0066	.0086	.0108	.0135	.0165	.0200	.0239	.0333	.0447
		.0030	.0042	.0055	.0071	.0090	.0112	.0138	.0167	.0199	.0277	.0373
$d = \frac{3}{4}$ in.	D_1	3.00	3.25	3.50	3.75	4.00	4.25	4.50	4.75	5.00	5.50	6.00
	P	4.418	3.976	3.615	3.313	3.058	2.840	2.051	2.485	2.339	2.093	1.893
	s_1	.0028	.0038	.0051	.0066	.0084	.0105	.0129	.0157	.0189	.0264	.0356
		.0024	.0033	.0044	.0057	.0072	.0090	.0111	.0135	.0162	.0226	.0305
		.0020	.0027	.0036	.0047	.0060	.0075	.0093	.0113	.0135	.0188	.0254
$d = \frac{7}{8}$ in.	D_1	3.50	3.75	4.00	4.25	4.50	4.75	5.00	5.25	5.50	6.00	6.50
	P	6.013	5.490	5.051	4.676	4.354	4.073	3.826	3.607	3.413	3.080	2.806
	s_1	.0021	.0027	.0035	.0045	.0055	.0067	.0081	.0097	.0115	.0156	.0207
		.0018	.0024	.0030	.0038	.0047	.0058	.0070	.0083	.0098	.0134	.0177
		.0015	.0020	.0025	.0032	.0039	.0048	.0058	.0069	.0082	.0112	.0148
$d = 1$ in.	D_1	3.50	3.75	4.00	4.25	4.50	4.75	5.00	5.25	5.50	6.00	6.50
	P	9.425	8.568	7.854	7.250	6.732	6.283	5.890	5.544	5.236	4.712	4.284
	s_1	.0012	.0016	.0021	.0026	.0033	.0041	.0049	.0059	.0071	.0097	.0129
		.0010	.0014	.0018	.0023	.0028	.0035	.0043	.0051	.0061	.0083	.0111
		.0008	.0011	.0015	.0019	.0023	.0029	.0035	.0043	.0051	.0069	.0092

D_1 = *outside* diameter of coil;
d = diameter of steel wire or rod;
s_1 = deflection of *one* coil at a load of 100 pounds;
P = safe load in pounds; also the load by which a specified deflection is to be effected;
S = fiber stress in pounds per square inch = 60,000;
E_s = modulus of torsional elasticity.

Comparing the values of D_1 and s_1 with those of D and s used previously, it is seen that

$$D_1 = D + d,$$

also that the *total* deflection s for a given load is

$$s = \frac{n s_1 P}{100} \qquad (1)$$

and

$$n = \frac{100 s}{s_1 P} \qquad (2)$$

Table III may also be used in calculations for helical springs made of *square* steel. For square steel, the values given in the table for P must be multiplied by 1.2, and the values for s_1 by .59. Three values of s_1 are given for each separate value of d corresponding to the following values of E_s:

$$E_s = 10{,}000{,}000$$
$$E_s = 12{,}000{,}000$$
$$E_s = 14{,}000{,}000$$

43. Application of Table III.—The use of Table III may be illustrated by the following examples:

EXAMPLE 1.—What is the safe load for a spring made of $\frac{5}{8}$-inch round steel, if the outside diameter D_1 is 3 inches?

SOLUTION.—Descend along the first column of Table III until the value $d = \frac{5}{8}$ in. is found. In the second column, an adjacent letter D_1 is found. Proceed to the right along the line headed by D_1 until the value 3 in. is found, and below this, in the line P, is the number 2,422, which is the safe load for the spring.

If this spring is to show a deflection of 2 in. for a load of 2,000 lb., ascertain the number of coils required, taking the average value for E_s as 12,000,000. Below 2,422 in the table are three other numbers headed by the letter s_1; the second of these is $s_1 = .0058$, which corresponds to $E_s = 12{,}000{,}000$. Applying formula **2**, Art. **42**,

$$n = \frac{100 \times 2}{.0058 \times 2{,}000} = 17.24, \text{ or } 17, \text{ coils, nearly.} \quad \text{Ans.}$$

EXAMPLE 2.—A spring having an outside diameter of 4 inches is made of 10 coils of $\frac{1}{2}$-inch round steel. Will an extension of 4 inches strain the spring?

SOLUTION.—From Table III, the greatest permissible load P is 841 lb. If $E_s = 12{,}000{,}000$, then $s_1 = .0457$. Inserting these and the other known data in formula **1**, Art. **42**,

$$s = \frac{10 \times .0457 \times 841}{100} = 3.84 \text{ in.}$$

As the spring is extended 4 in., it is supporting an excessive load and will therefore be strained. Ans.

EXAMPLE 3.—Find the safe load for a spring made of $\frac{1}{2}$-inch square steel and having an outside diameter of 3 inches.

SOLUTION.—From the table, $P = 1{,}178$ lb. for round steel, but, as stated in Art. **42**, for square steel, this value should be multiplied by 1.2; hence,

$$P = 1{,}178 \times 1.2 = 1{,}414 \text{ lb.} \quad \text{Ans.}$$

EXAMPLE 4.—Find the deflection of the spring in the last example at a load of 1,000 pounds, if it has 15 coils.

SOLUTION.—For square steel, $s_1 = .59\ s_1$, when the latter value of s_1 is that found in Table III for round steel. Applying formula **1**, Art. **42**,

$$s = \frac{15 \times .59 \times .0167 \times 1{,}000}{100} = 1.48 \text{ in.} \quad \text{Ans.}$$

SPIRAL SPRINGS

44. A **spiral spring** consists of coils wound spirally on one another in a plane at right angles to the axis of the spring, as shown in Fig. 26. Springs of this class are properly termed *flat spiral springs* to distinguish them from *conical spiral springs*, in which the coils advance longitudinally as well as radially. A railroad buffer spring is an example of the latter class.

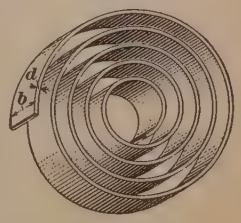

FIG. 26

45. Flat Spiral Springs.—In **a flat spiral spring**, the bending moment is equal to

$$M = Wr = \frac{S_t I}{c}, \qquad (1)$$

in which W = load, in pounds;

r = lever arm of load, in inches, considering center of spring as one end of arm (see Fig. 27);

S_s = safe outer fiber stress, in pounds per square inch;

d = thickness of bar, in inches;

b = width of bar, in inches.

For a bar or wire of rectangular cross-section, $I = \frac{b\,d^3}{12}$ and $c = \frac{d}{2}$. Inserting these values in formula **1**,

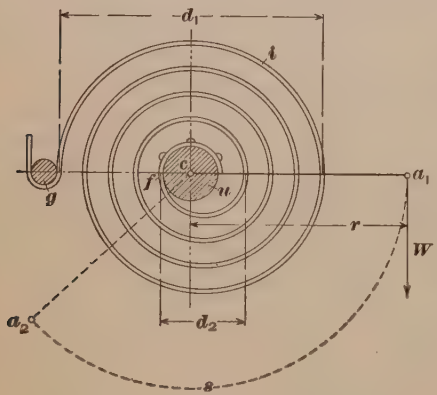

FIG. 27

$$Wr = \frac{S_s I}{c} = \frac{S_s b\,d^2}{6}$$

and

$$W = \frac{S_s b\,d^2}{6\,r} \qquad (2)$$

The parts of the spring that are fastened to the shaft, or that are wound around it so as to be in contact with it, will be unable to bend and are therefore inactive. In Figs. 27 and 28, the active part of the spring is supposed to begin at point f, and the length of the spring or the number of active coils are to be counted from this point.

Let l = developed length, in inches, of all the active coils;

s = distance, in inches, moved through by the point a_1, the point at which the load W is applied (Fig. 27);

$E_t = E_c$ = modulus of elasticity = 30,000,000 for steel;

e = angular deflection in degrees of point a_1.

Then, the following formulas may be evolved:

$$s = \frac{12\,W l\,r^2}{E_t b\,d^3} \qquad (3)$$

and the angular deflection

$$e = \frac{57.3\ s}{r} \qquad (4)$$

In explanation of Fig. 27, it should be noted that when considering the deflection of the spring i, reference is had to the relative motion between the ends of the spring. In this instance, one end is attached to the stationary stud g, and the other end to the shaft u, which may be turned by the force W acting on the lever arm $r = c\,a_1$. The point a_1 is supposed to move into the position a_2, while the force W acts at right angles to the lever arm $c\,a_1 = c\,a_2$. The linear deflection s is equal to the length of the arc $a_1\,a_2$, while the angular deflection e corresponds to the angle $a_1\,c\,a_2$.

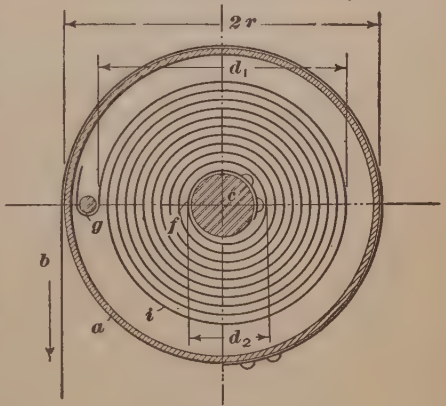

FIG. 28

46. The length l in formula **3**, Art. **45**, is found by means of the following formula:

$$l = \pi n \frac{d_1 + d_2}{2} \qquad (1)$$

in which d_1 = diameter of outside coil (Fig. 27), in inches;
d_2 = diameter of inside coil, in inches;
n = number of active coils.

In Fig. 27, for instance, there are four active coils.

If the value of l in formula **1** is inserted in formula **3**, Art. **45**, then

$$s = \frac{18.85\,W r^2 n}{E_t\,b\,d^3}(d_1 + d_2) \qquad (2)$$

EXAMPLE.—A pulley a 18 inches in diameter revolves loosely on the stud c, Fig. 28, and supports at intervals a load of 400 pounds by means of the strap b. The rotation of the pulley is opposed by a spiral spring i made of a steel band 3 inches wide and $\frac{3}{8}$ inch thick. One end of the spring is attached to the stud g on the pulley, and the other end to the stud c. If the diameters of the outer and inner coils of the

spring are 16 and 4 inches, respectively, and there are 10 active coils, what will be the linear and angular motion of a point on the pulley rim when the load is acting?

SOLUTION.—It is first necessary to ascertain whether the spring will safely support the load of 400 lb. From formula **2**, Art. **45**, $W = \frac{S_s b d^2}{6r}$. The diameter of the pulley is $2r = 18$ in.; hence, $r = 9$ in. Inserting this value of r in the formula and using a value for S_s of 60,000,

$$W = \frac{60{,}000 \times 3 \times .375^2}{6 \times 9} = 469 \text{ lb.}$$

The spring therefore has sufficient strength to support the load of 400 lb.

From formula **2** of this article,

$$s = \frac{18.85 \times 400 \times 9^2 \times 10}{30{,}000{,}000 \times 3 \times .375^3} \times (16 + 4) = 25.7 \text{ in.} \quad \text{Ans.}$$

From formula **4**, Art. **45**,

$$e = \frac{57.3 \times 25.7}{9} = 163.6°. \quad \text{Ans.}$$

T4-AKA-400